Business Cycles in the Equilibrium Model of Labor Market Search and Self-Insurance*

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Abstract

I introduce risk-aversion, labor-leisure choice, capital, individual productivity shocks, and market incompleteness to the standard model of labor search and matching and investigate the model's cyclical properties. I find that the model can generate the observed large volatility of unemployment and vacancies with a reasonable replacement rate of unemployment insurance benefits of 64 percent. Labor-leisure choice plays a crucial role through additional utility from leisure when unemployed and further amplification from adjustments of hours worked. On the other hand, the borrowing constraint or individual productivity shocks do not significantly affect the cyclical properties of unemployment and vacancies.

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1 Introduction

The main purpose of the paper is to embed the standard labor search and matching model into the standard business cycle model with risk-averse workers, labor-leisure choice, capital, individual productivity shocks, and market incompleteness, and re-examine the puzzle presented by Shimer (2005). The Mortensen-Pissarides search and matching model has become the standard theory of equilibrium unemployment (Mortensen and Pissarides (1994), Pissarides (2001)). However, Shimer (2005) pointed out that the Mortensen-Pissarides model with labor productivity shocks cannot replicate the volatility of unemployment and vacancies observed in the postwar U.S. data. The volatilities of unemployment and vacancies in the Mortensen-Pissarides model with Shimer's (2005) calibration strategy are about one-tenth of their volatilities in the U.S. data.

In the baseline model that I construct in this paper, workers can accumulate capital but the market is incomplete; workers cannot write a state-contingent contract to insure away the individual productivity shocks and unemployment shocks, but they can self-insure by accumulating capital. In addition, workers in the model choose how many hours to work and how many hours to enjoy leisure. By comparing the properties of the baseline model and the Mortensen-Pissarides model with Shimer's (2005) calibration, I will explore the role played by the two additional features, labor-leisure choice and market incompleteness, in shaping the cyclical properties of unemployment and vacancies.

The second purpose of the paper is to compare the business cycle properties of the baseline model with those of the standard real business cycle (RBC) model. The current model can also be considered a standard real business cycle model that is extended by incorporating search and matching frictions in the labor market. Since the current model generates fluctuations in employment (extensive margin) and average hours worked (intensive margin), the model has the capacity to match a variety of cyclical properties of U.S. business cycles, especially those associated with the labor market.

There are four main findings. First, the baseline model successfully replicates the observed large volatility of unemployment and vacancies. In other words, the model has a strong amplifying mechanism. As will be shown, the intuition is similar to how the Mortensen-Pissarides model calibrated by Hagedorn and Manovskii (2008a) successfully generates a large volatility of unemployment and vacancies. However, the baseline model developed in the current paper achieves the strong amplification with a reasonably low replacement rate for unemployment insurance benefits of 64 percent, while the calibrated value of the replacement rate in Hagedorn and Manovskii (2008a) is 95.5 percent.

My second finding is that labor-leisure choice plays a key role for the main result in two ways. Utility from leisure narrows the gap between the value of being unemployed and that of being employed, even with the reasonable replacement rate of 64 percent. The small gap between the two values is the key property for the model to exhibit a strong amplification, as Hagedorn and Manovskii (2008a) argue using the standard Mortensen-Pissarides model. Naturally, the model's strong amplification vanishes when the utility from leisure is turned off. Without utility from leisure, a very high replacement rate of unemployment insurance benefits like the one used by Hagedorn and Manovskii (2008a) (95.5 percent) is needed for a strong amplification. In other words, the model implies that utility from leisure is equivalent to approximately 30 percent of the replacement rate, which is the difference between 95.5 percent and 64 percent. Labor-leisure choice also helps to generate strong amplification through the intensive margin of labor supply adjustments over the business cycles.

Third, I find that the cyclical properties of unemployment and vacancies are affected little by relaxing the borrowing constraint or strengthening or shutting down the uninsured individual productivity shocks. In the canonical incomplete-markets model, even if workers are not allowed to trade state-contingent securities, workers save and self-insure sufficiently so that the incomplete-markets model with a borrowing constraint and individual productivity shocks behaves like its complete-markets counterpart at the aggregate level. Under this situation, a relaxed borrowing constraint or changing volatility of the individual productivity shocks has little effect on the cyclical properties of the model. This intuition carries over to the cyclical properties of unemployment and vacancies studied in this paper.

Fourth, I find that the model shows improvement in replicating many cyclical properties of the U.S. labor market that the standard RBC models or the previous RBC models with labor market frictions (Andolfatto (1996) and Merz (1995)) cannot produce. This is mainly due to the labor market frictions and the strong amplification mechanism of the model. In particular, compared with the previous RBC models with labor market frictions, the baseline model in the current paper generates a stronger volatility of unemployment and vacancies, and a strong negative correlation between the two.[2] On the other hand, the cyclical properties of the baseline model with respect to consumption and investment are similar to those of the standard real business cycle model. This similarity echoes the well-established findings of Krusell and Smith (1998) that incomplete-markets models exhibit cyclical properties of aggregate variables similar to those in their complete-markets counterparts.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 summarizes the cyclical properties of the U.S. economy. The model's performance is measured by the extent to which the model can replicate the properties. In Section 4, the baseline model is presented. Section 5 discusses the calibration of the model, and Section 6 offers a brief discussion of the solution methods. Section 7 presents the main results of the paper and discusses them. Section 8 concludes. Appendix A gives details of the computational methods.

2 Related Literature

The current paper is built on two groups of the literature. The first group is trying to solve the puzzle presented by Shimer (2005) mainly by modifying the basic Mortensen-Pissarides model. The second group extends the standard real business cycle model by introducing labor market search and matching. To the best of my knowledge, the model developed in the current paper is one of the first models that combines the incomplete-markets real business cycle model with labor market search and matching in the general equilibrium framework.

The first group of research starts with Shimer (2005), who finds that the standard Mortensen-Pissarides model of labor market search and matching with aggregate shocks to labor productivity and Nash bargaining cannot generate an observed large volatility of unemployment and vacancies.

[2] The stronger negative correlation between unemployment and vacancies is also due to the timing assumption, by which the employment/unemployment adjustment occurs in the same period as the vacancy posting. On the other hand, Andolfatto (1996) assumes that it takes one model period (a quarter) for unemployment to respond to the vacancy posting.

Hall (2005) argues that the problem lies in the Nash bargaining assumption in wage setting. He points out that if there is stickiness in real wage setting, instead of real wage elastically responding to changes in productivity, the labor search and matching model can produce a large volatility of unemployment and vacancies. If real wages are sticky, firms' profits respond more to the changes in productivity, which leads to the larger volatility of vacancy postings and, eventually, unemployment.

Hagedorn and Manovskii (2008a) argue that the problem does not lie in the model itself but in the way the model is calibrated. To make their point, they use the same Mortensen-Pissarides model that Shimer (2005) uses, but employ an alternative calibration strategy and show that the same model can generate the degree of amplification as strong as that observed in data. The key feature of their calibration strategy is to pin down the replacement rate of unemployment insurance benefits and the parameter representing the worker's bargaining weight to match two observed targets, namely, the average labor market tightness and the elasticity of wage with respect to labor productivity.

Costain and Reiter (2006) propose that cohort-specific technology shocks can improve the model's performance. Their model generates a higher volatility of unemployment and vacancies because the number of vacancies posted is sensitive to the value of *new* matches, not to the value of *existing* matches, and the cohort-specific technology shocks only affects the value of new matches. Pries (2007) introduces heterogeneity across workers' productivity and shows that the composition effect associated with the productivity of workers helps to generate a high volatility of unemployment and vacancies. Pissarides (2009) argues that the standard search model can generate a large unemployment volatility with sunk fixed matching costs.

In the second group of research related to the current paper, Andolfatto (1996) and Merz (1995) are the seminal works. Both extend the standard complete-markets RBC model by introducing Mortensen-Pissarides type labor market frictions and point out that the performance of real business cycle models can be improved by incorporating labor market frictions in the form of labor market search and matching. However, neither is very successful in generating the observed cyclical properties of unemployment and vacancies, in particular, their high volatility and the negative correlation between them. The novelty of the current paper is that the model developed here successfully replicates these properties of the data. In addition, the model developed in this paper features market incompleteness and thus can be used to investigate the importance of market incompleteness in shaping the cyclical properties of the model.

Within the complete-markets framework, Gertler and Trigari (2009) extend the models of Andolfatto (1996) and Merz (1995) by introducing staggered multi-period wage contracting and show that their model exhibits a strong amplification, along with various improvements in replicating U.S. business cycle properties. Den Haan et al. (2000) extends the real business cycle model with labor market frictions by introducing endogenous match destruction. More recently, Jung (2006) constructs a version of the real business cycle model with labor market search and marching and a representative family and shows that the model can replicate the volatility of unemployment and vacancies with reasonable parameter values.

Regarding the incomplete-markets framework, an innovative work by Gomes et al. (2001) combines the incomplete-markets real business cycle model with labor market search and successfully generates a large volatility of unemployment. However, the model is different from the standard Mortensen-Pissarides model; there is no matching or unfilled vacancy in the model. More recently, Krusell et al. (2010), Bils et al. (2009), and Shao and Silos (2007) construct models that feature saving, incomplete markets, and labor market frictions as in the current paper. However, none of the papers has a worker's labor-leisure choice. Therefore, none can analyze the importance of the labor-leisure choice in shaping the cyclical properties of the model or analyze the cyclical properties of the intensive and extensive margins of labor supply adjustments, as I do in the current paper. I also investigate the importance of the borrowing constraint and the individual productivity shocks in shaping the cyclical properties of the incomplete-markets model. More specifically, Krusell et al. (2010) use the model to conduct a welfare analysis of the unemployment insurance program. They do not have heterogeneity in terms of individual productivity of workers. In other words, the version of the model developed here without the labor-leisure choice and heterogeneous productivity of workers is similar to their model.[3] Bils et al. (2009) do have heterogeneity with respect to match quality, and they investigate the implications of the heterogeneity with respect to both match quality and asset holding on the cyclical properties of the model. Shao and Silos (2007) study the implications of wealth distribution on the cyclical properties. They do not include heterogeneity of individual productivity, either.

The way in which market incompleteness is introduced in the current paper follows Huggett (1993) and Aiyagari (1994); workers in the model face idiosyncratic uncertainty and can only self-insure by saving. Krusell and Smith (1998) introduce aggregate uncertainty into the model by Aiyagari (1994). Regarding the solution method, the approximate equilibrium developed by Krusell and Smith (1998) is employed. The model developed in this paper can be considered an extension of their model in the sense that the formation and resolution of matches is endogenous in the current model, while it is exogenous in Krusell and Smith's (1998) model.

3 Facts

Table 1 summarizes the business cycle facts of the U.S. economy between 1951 and 2009. All data are quarterly. The cyclical properties are computed using the log of the data after filtering with the Hodrick-Prescott (H-P) filter. Following the standard practice in the business cycle literature for quarterly data, a smoothing parameter of 1600 is used.[4]

The first block of the table (the first three rows) contains the cyclical properties of output and its major components, namely, consumption and investment. Consumption in the table represents total consumption expenditure, which includes nondurable and durable consumption goods as well as services. The volatility of nondurable consumption goods is slightly lower than total consumption. Investment in the table includes both fixed investment, which consists of residential and nonresidential investment, and change in private inventories.

The second block of the table (the fourth to the seventh rows) contains the cyclical properties of unemployment, vacancies, their ratio (which in the literature is called *market tightness*) and the probability that an unemployed worker finds a job. The data on the number of vacancies are

[3] In the previous version of the current paper, collective bargaining was used instead of the standard bilateral bargaining, which was used in their paper.

[4] Shimer (2005) uses a smoothing parameter of 10^5. According to Hornstein et al. (2005) (see Table 1), using the smoothing parameter of 10^5 instead of the standard value of 1600 does not generate a substantial difference in terms of the volatility of the important labor market variables relative to the volatility of labor productivity.

Table 1: Cyclical properties of U.S. economy: 1951:1-2009:3[1,2]

Variable	SD%	Relative SD%[3]	Auto-corr	Cross-correlation of output with x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}
Output	1.58	1.00	0.84	0.59	0.84	1.00	0.84	0.59
Consumption[4]	1.27	0.80	0.83	0.69	0.83	0.85	0.69	0.46
Investment[5]	7.34	4.64	0.80	0.56	0.76	0.88	0.72	0.48
Unemployment rate (u)	12.95	8.19	0.87	−0.36	−0.64	−0.84	−0.85	−0.71
Vacancies (v)	14.10	8.91	0.91	0.56	0.78	0.89	0.82	0.65
Market tightness (v/u)	26.44	16.71	0.90	0.48	0.73	0.89	0.85	0.69
Job finding rate	8.38	5.30	0.80	0.43	0.67	0.82	0.82	0.66
Aggregate hours[6]	1.58	1.00	0.88	0.39	0.66	0.87	0.89	0.76
Employment[6]	1.05	0.66	0.87	0.34	0.60	0.80	0.85	0.76
Average weekly hours[7]	0.51	0.32	0.77	0.58	0.69	0.72	0.54	0.31
Labor share[8]	0.77	0.49	0.75	−0.34	−0.30	−0.19	0.07	0.27
Output per hour	0.80	0.50	0.68	0.40	0.35	0.26	−0.09	−0.34
Wage[9]	0.90	0.57	0.74	0.07	0.05	0.07	−0.02	−0.07
Output per worker	0.97	0.61	0.70	0.60	0.72	0.76	0.45	0.15
Compensation per worker[10]	1.00	0.63	0.74	0.32	0.47	0.60	0.35	0.18

[1] Source: BEA (output and its components), BLS (labor market-related data), Conference Board (help-wanted advertising index used as the proxy for the number of vacancies).
[2] All data are quarterly between 1951:1 and 2009:3. Logs of the data are filtered using the H-P filter with a smoothing parameter of 1600.
[3] Relative to the standard deviation of output.
[4] Including nondurable and durable consumption goods as well as services.
[5] Including residential and nonresidential investment as well as changes in private inventories.
[6] Based on the establishment survey.
[7] Based on the household survey.
[8] Computed using the method proposed by Hansen and Prescott (1995).
[9] Computed as output per hour multiplied by labor share.
[10] Computed as output per worker multiplied by labor share.

constructed in the same way as in Shimer (2005); the help-wanted advertising index compiled by the Conference Board is used as a proxy for the number of vacancies. Monthly job finding probability is constructed in the way suggested by Shimer (2005). For monthly data series, the average of the monthly data in each quarter is used. The properties of these data are important because these are the ones that Shimer (2005) claims the standard Mortensen-Pissarides model fails to replicate.

The third block of the table (the eighth to the tenth rows) is associated with the total labor supply, and adjustments in the extensive and intensive labor supply. The fourth and last block of the table (the last five rows) is associated with productivity and wages.

The key features of the cyclical properties of the U.S. economy presented in Table 1 are summarized as follows. The performance of the model constructed in the current paper will be judged by how well the model replicates these cyclical properties shown in Table 1, in particular, the cyclical properties of unemployment and vacancies.

1. **(Output)** Volatility of detrended output is 1.58 percent. Output is strongly persistent, with an autocorrelation of 0.84.
2. **(Consumption)** Consumption is less volatile than output (consumption volatility is 0.8 times output volatility) and strongly procyclical.
3. **(Investment)** Investment is substantially more volatile than output (investment volatility is 4.6 times output volatility) and strongly procyclical.
4. **(Unemployment and vacancies)** The unemployment rate is about eight times as volatile as output and strongly countercyclical. The number of vacancies posted is about nine times as volatile as output and strongly procyclical.
5. **(Market tightness)** The market tightness (the ratio of vacancies over the unemployment rate) is about 17 times as volatile as output and strongly procyclical. What is implied is that the correlation between unemployment and vacancies is strongly negative (-0.91). This is typically shown as the Beveridge curve.
6. **(Job finding rate)** Job finding rate is about five times as volatile as output and strongly procyclical.
7. **(Aggregate hours)** Aggregate hours worked is as volatile as output, strongly procyclical, and, more important, lags output by one quarter.
8. **(Employment and average hours)** If the volatility of the aggregate hours is decomposed into the volatility of employment (extensive margin) and the volatility of hours per worker (intensive margin), the former accounts for two-thirds of the volatility of the aggregate hours.[5] The latter accounts for one-third. Employment lags the cycle by about one quarter. This translates into a strong correlation between output and lagged aggregate hours. Meanwhile, there is no lead or lag with respect to the average hours worked.
9. **(Labor share)** Labor share (share of total income earned by employees) is half as volatile as output and mildly countercyclical (correlation of -0.19).
10. **(Productivity and wages)** Productivity measured by output per hour is less volatile (about half) than output and weakly procyclical (correlation of 0.26). Wage measured by labor income per hour is also less volatile (relative standard deviation of 0.57) and close

[5] I use the employment data series based on the establishment survey. Hagedorn and Manovskii (2008b) point out that the employment data based on the household survey exhibit different cyclical properties.

to acyclical. Both output per worker and compensation per worker are less volatile than output (standard deviation of about 60 percent) and strongly procyclical.

4 Model

The model is best described as the incomplete-markets general equilibrium model of Krusell and Smith (1998), which itself is the model of Aiyagari (1994) with aggregate uncertainty, extended by incorporating labor market frictions in the form of the Mortensen-Pissarides model.

4.1 Preferences

Time is discrete. The economy is populated by a mass of infinitely lived workers and firms. The total measure of workers is normalized to unity. A worker maximizes his expected lifetime utility. The expected lifetime utility of a worker takes the following time-separable form:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \right\} \tag{1}$$

where β is the time discount factor, and $\mathbb{E}_0$ is an expectation operator with information available at period 0. c_t is the consumption of the worker in period t. h_t is the leisure time enjoyed in period t. The period utility function $u(c, h)$ is assumed to be strictly increasing and strictly concave with respect to each of c and h.

There are a large number of firms. Each firm is risk-neutral and maximizes the present value of its profit stream. The firms discount their future profits using the equilibrium real interest rate.

4.2 Endowment

A worker is endowed with capital a_0 in the initial period and one unit of time each period. Workers can use their time for either work or leisure. Hours for work and leisure in period t are denoted as ℓ_t and h_t, respectively. The following time constraint must be satisfied in any period:

$$h_t + \ell_t = 1 \tag{2}$$

4.3 Production Technology

In order to produce, a worker and a firm have to be matched. I call a matched worker employed ($e = 1$) and an unmatched worker unemployed ($e = 0$). A firm can also be either matched ($e = 1$) or unmatched ($e = 0$). Matched pairs have access to the following production technology:

$$Y_t = e^{z_t} F(K_t, L_t) \tag{3}$$

where K_t is capital input, L_t is labor input, Y_t is output, and e^{z_t} is total factor productivity (TFP). z has a finite support $Z = \{z_1, z_2, ..., z_{n_z}\}$ and follows a first-order Markov process with the transition probability $\pi^z_{zz'}$. It is assumed that the production function $F(K, L)$ satisfies constant returns to scale and is strictly increasing and strictly concave with respect to each input. Capital stock depreciates at a constant rate δ.

Labor input is measured by the efficiency unit. When a worker's current individual productivity is denoted by s_t, and the worker works for ℓ_t hours, the labor input is $s_t\ell_t$. s has a finite support $S = \{s_1, s_2, ..., s_{n_s}\}$ and follows a first-order Markov process with the transition probability $\pi^s_{ss'}$. Notice that s is not match specific.

4.4 Job Turnover Technology

Workers without a job search for a job. Matching is random. There is no cost for job search. There is no search intensity decision. The probability of finding a job is the same across all workers who are searching for a job. Unmatched firms search for a worker by posting a vacancy. The cost of posting a vacancy is represented by a parameter κ. The probability of finding a worker is the same across all firms that are searching for a worker.

Let the total number of workers that are employed and unemployed in period t be denoted as N_t and U_t, respectively. Since the total number of workers is normalized to one, the number of employed workers N_t can be expressed as $N_t = 1 - U_t$. Let V_t be the total number of vacancies posted by unmatched firms in period t.

For a matched pair, there is a constant probability λ that the match is dissolved, meaning that the measure of matches that dissolve can be expressed as λN_t. There is no on-the-job search.

I assume that a worker whose match is just dissolved can immediately start searching for a new job. It implies that a worker whose match is dissolved but who immediately finds the next job does not experience an unemployment spell. This timing assumption is needed to achieve a proper calibration; in the baseline calibration, a period will be set as a quarter, which is the choice of the standard real business cycle models. However, if a worker in a dissolved match is forced to stay unemployed for at least one period, the average duration of unemployment spells will be longer than one model period (which is one quarter). It is impossible to reconcile the implied average duration of unemployment spells (longer than a quarter) of the model with the empirical quarterly separation probability of 10 percent and the average unemployment rate of 5.67 percent. In order to distinguish between the number of unemployed workers and the number of workers that are searching for a job, I use S_t for the measure of workers who are searching for a job. By construction, the following holds:

(4) $$S_t = U_t + \lambda N_t$$

It is assumed that the number of new matches M_t is a function of the total number of workers searching for a job (S_t) and the total number of vacancies posted (V_t). Formally, M_t is characterized as follows.

(5) $$M_t = \min(f_m(S_t, V_t), S_t, V_t)$$

The calibration makes sure that $M_t = f_m(S_t, V_t) < min(S_t, V_t)$ always holds. Therefore, I call the function $f_m(S, V)$ the aggregate matching function.

4.5 Market Structure

Capital is rented in the competitive market, with the rental price r_t, which is equal to the marginal product of capital net of depreciation in equilibrium. Labor can be supplied only in a matched pair. All firms are assumed to be owned by all workers jointly. As a result, the sum of firms' profits in each period, net of total costs for posting vacancies, is shared by the owners (workers) equally as a lump-sum transfer d_t.

Remember that production takes place in each of the matched pairs. In other words, there is no aggregate production function. However, because the capital rental market is competitive, all producing pairs have to offer the same rate of return of capital in equilibrium. Since the production technology is assumed to exhibit constant returns to scale, this further implies that the capital-labor ratio will be equated across all the worker-firm pairs in equilibrium. Therefore, one can think of a stand-in aggregate production function with the aggregate capital stock K_t and aggregate labor supply L_t. The rate of return of capital r_t and the marginal product of labor p_t in all the matched pairs will be the same as the respective marginal products implied by the aggregate production function.

Markets are incomplete in the sense that workers are not allowed to trade securities contingent on the state of the world or individual characteristics and thus to insure against aggregate or individual shock. Against the unemployment shock, partial insurance is provided by the government in the form of unemployment insurance, which is discussed below. Apart from the publicly provided unemployment insurance, all that workers can do is to save in the form of capital and self-insure. Moreover, workers' capital holding is constrained by the lower bound $\underline{a}$. In the baseline case, I assume $\underline{a} = 0$; i.e., workers cannot borrow. In one of the experiments in Section 7, I will investigate the implications of a relaxed borrowing constraint.

4.6 Unemployment Insurance

The government runs an unemployment insurance program. An unemployed worker receives the unemployment insurance benefits of b_t. As will be specified, b_t depends on the individual productivity of a worker. In order to finance the program, the government taxes the labor income of all employed workers using a proportional tax of rate τ.

In each period, the balance of the program might be either positive or negative, depending mainly on the number of workers employed and unemployed. The balance of the program will be equally distributed across all workers in a lump-sum transfer every period. In particular, if the total contribution is larger (smaller) than the total benefits, the difference is distributed to all workers as a lump-sum transfer (tax). t_t denotes the per capita balance of the unemployment insurance program in period t and naturally takes a positive (negative) value when the total contribution is larger (smaller) than the total benefits.

4.7 Bargaining

The hours worked by the worker, ℓ_t, and the surplus-sharing rule, w_t (which represents the proportion of the total surplus allocated to the worker), are jointly determined as the solution of the generalized Nash bargaining. It implies that the hours ℓ_t maximize the joint surplus. More specifics of the bargaining will be discussed in Section 4.11, after characterizing the values of the worker and the firm.

4.8 Recursive Formulation

Equilibrium is defined recursively. From now on, time subscripts are dropped and variables in the next period are denoted by a prime. A worker can be characterized by a triplet (e, s, a), where $e \in E = \{0, 1\}$ is the employment status, $s \in S = \{s_1, s_2, ..., s_{n_s}\}$ is the individual productivity shock, and $a \in A = [\underline{a}, \infty)$ is the capital holding. $e = 0$ denotes unemployed or unmatched, while $e = 1$ denotes employed or matched. For ease of notation, I define x as a probability measure over $\mathcal{X}$, which is a σ-algebra generated by the set $X \equiv E \times S \times A$. x represents a type distribution of workers. Before proceeding further, it is useful to describe below the sequence of events in a period. Notice that since z follows a first-order Markov process, the current z is sufficient to predict the future realizations of z.

1. Beginning of period. Shocks (z, s) are realized. The aggregate state is $(z, \widetilde{x})$, where $\widetilde{x}$ is the type distribution before matches are destroyed and created.
2. Matches are destroyed and created. Specifically, $\lambda N(\widetilde{x})$ matches are destroyed. $S(\widetilde{x})$ is the measure of workers searching for a job. Remember that $\lambda N(\widetilde{x})$ workers can immediately start searching. $\widetilde{V}(z, \widetilde{x})$ vacancies are posted. $M(z, \widetilde{x})$ matches are created.
3. The aggregate state variable is (z, x), where x is the type distribution after matches are destroyed and created.
4. Bargaining takes place; $\ell(z, x, s, a)$ and $w(z, x, s, a)$ are determined.
5. Firms rent capital and produce. The rate of return to capital and labor productivity are $r(z, x)$ and $p(z, x)$, respectively.
6. Given the bargaining outcome, prices, dividend income from firms $d(z, x)$, unemployment insurance benefit $b(s, a)$, and the lump-sum transfer from the unemployment insurance program, $t(z, x)$, workers decide how much to consume and save.
7. $\widetilde{x}'$ is the type distribution of workers after the consumption-saving decision is made, and the new individual productivity shocks are drawn.

Notice that employment, unemployment and vacancies adjust *before* production takes place. On the other hand, in the related model by Andolfatto (1996), it is assumed that they move *after* production takes place. Such a timing assumption implies that there is at least a lag of one period between production and match creation and destruction, which artificially lags employment and unemployment relative to output. To the contrary, in the current set-up, such a lag is avoided. I will come back to the implication of this timing assumption when I evaluate the cyclical properties of the model.

Using the aggregate state, the total measure of workers searching for a job, S, the number of matches, M, the aggregate (or average, because the size of the total population is normalized to unity) capital stock, K, and the aggregate labor supply, L, can be characterized by the following functions:

$$S(\widetilde{x}) = \int_X \mathbb{1}_{e=0}\, d\widetilde{x} + \lambda \int_X \mathbb{1}_{e=1}\, d\widetilde{x} \tag{6}$$

$$M(z, \widetilde{x}) = f_m(S(\widetilde{x}), \widetilde{V}(z, \widetilde{x})) \tag{7}$$

$$K(x) = \int_X a\, dx \tag{8}$$

$$L(z, x) = \int_X s\ell(z, x, s, a)\, dx \tag{9}$$

where $\mathbb{1}_{condition}$ is an indicator function that takes the value of 1 if the *condition* is true, and 0 otherwise. $\widetilde{V}(z, \widetilde{x})$ is the total number of vacancies posted, which is determined by the optimal entry decision of unmatched firms and thus is a function of aggregate states. I will discuss the determination of $\widetilde{V}(z, \widetilde{x})$ later. Furthermore, these functions can be used to construct functions of matching probability for a worker searching for a job (f_w) and for an unmatched firm (f_j) as follows:

$$(10) \quad f_w(z, \widetilde{x}) = \frac{M(z, \widetilde{x})}{S(\widetilde{x})}$$

$$(11) \quad f_j(z, \widetilde{x}) = \frac{M(z, \widetilde{x})}{\widetilde{V}(z, \widetilde{x})}$$

Finally, I define below the two transition functions for the probability measure, corresponding to the type distribution of workers before ($\widetilde{x}$) and after (x) matches are destroyed and created:

$$(12) \quad \widetilde{x}' = \widetilde{f}_x(z, x)$$

$$(13) \quad x = f_x(z, \widetilde{x})$$

4.9 Worker's Problem

The problem of a worker is defined recursively in this section. Given the law of motion of the type distribution, $\widetilde{f}_x(z, x)$ and $f_x(z, \widetilde{x})$, and functions for the real interest rate, $r(z, x)$, labor productivity, $p(z, x)$, dividends, $d(z, x)$, worker's share of surplus, $w(z, x, s, a)$, hours worked, $\ell(z, x, s, a)$, unemployment insurance benefit, $b(s, a)$, lump-sum transfer from the unemployment insurance program, $t(z, x)$, and the job finding probability, $f_w(z, \widetilde{x})$, the worker's problem can be recursively formulated as follows:

$$(14) \quad W(z, x, e, s, a) = \max_{c \geq 0, a' \geq \underline{a}} \left\{ u(c, h) + \beta \sum_{z'} \sum_{s'} \sum_{e'} \pi^z_{zz'} \pi^s_{ss'} P^W_{ee'} W(z', x', s', e', a') \right\}$$

subject to

$$(15) \quad c + a' = y + (1 + r(z, x))a + d(z, x) + t(z, x)$$

$$(16) \quad h = \begin{cases} 1 - \ell(z, x, s, a) & \text{if } e = 1 \text{ (employed)} \\ 1 & \text{if } e = 0 \text{ (unemployed)} \end{cases}$$

$$(17) \quad y = \begin{cases} p(z, x) s \ell(z, x, s, a) w(z, x, s, a)(1 - \tau) & \text{if } e = 1 \text{ (employed)} \\ b(s, a) & \text{if } e = 0 \text{ (unemployed)} \end{cases}$$

$$(18) \quad x' = f_x(z', \widetilde{f}_x(z, x))$$

where

$$\begin{aligned} P^W_{10} &= \lambda(1 - f_w(z', \widetilde{f}_x(z, x))) \\ P^W_{11} &= 1 - P^W_{10} \\ P^W_{01} &= f_w(z', \widetilde{f}_x(z, x)) \\ P^W_{00} &= 1 - P^W_{01} \end{aligned}$$

Let $g_a(z,x,e,s,a)$, $g_c(z,x,e,s,a)$ be the optimal decision rules for saving and consumption, respectively, associated with the optimal value function that solves the problem above.

4.10 Firm's Problem

I will first characterize the problem of unmatched firms and go on to characterize the problem of matched firms. An unmatched firm can enter the market by paying a fixed cost κ and posting a vacancy. With probability $f_j(z,\widetilde{x})$, a firm that enters the market will be matched and start producing later in the same period. Formally, the value of an unmatched firm at the beginning of period, $\widetilde{J}(z,\widetilde{x})$ can be characterized by the following Bellman equation:

$$(19)\quad \widetilde{J}(z,\widetilde{x}) = -\kappa + \frac{f_j(z,\widetilde{x})}{S(\widetilde{x})}\int_X \left[\mathbb{1}_{e=0}J(z,x,s,a) + \mathbb{1}_{e=1}\lambda J(z,x,s,a)\right] d\widetilde{x} + (1-f_j(z,\widetilde{x}))\sum_{z'}\pi^z_{zz'}\frac{\widetilde{J}(z',\widetilde{x}')}{1+r(z',x')}$$

Notice two things. First, the second term of (19) contains two parts: one associated with those who were unemployed at the beginning of the period ($e=0$) and the other associated with those were employed at the beginning of the period but just lost their job ($e=1$). Second, the last term is associated with the case where the firm is not matched with a worker.

Assuming free entry, firms keep entering the market until the value of entering the market is pushed down to zero so that a firm is indifferent between entering the market and not entering at the margin. Formally, free entry implies:

$$(20)\quad \widetilde{J}(z,\widetilde{x}) = 0$$

Combining the free-entry condition (20) with the Bellman equation of an unmatched firm (19) yields:

$$(21)\quad \kappa = \frac{f_m(S(\widetilde{x}),V(z,\widetilde{x}))}{V(z,\widetilde{x})\ S(\widetilde{x})}\int_X \left[\mathbb{1}_{e=0}J(z,x,s,a) + \mathbb{1}_{e=1}\lambda J(z,x,s,a)\right] d\widetilde{x}$$

This equation implicitly characterizes the number of vacancies (number of firms entering the market) $\widetilde{V}(z,\widetilde{x})$. I also use $V(z,x)$ as the forecasted number of vacancies posted, conditional on state (z,x).[6] With $V(z,x)$, the lump-sum dividend $d(z,x)$ is characterized as follows:

$$(22)\quad d(z,x) = -\kappa V(z,x) + \int_X \mathbb{1}_{e=1}j(z,x,s,a)dx$$

Given the law of motion for x, $\widetilde{f}_x(z,x)$ and $f_x(z,\widetilde{x})$, optimal saving function, $g_a(z,x,e,s,a)$, and functions for the real interest rate, $r(z,x)$, the worker's share of the surplus, $w(z,x,s,a)$,

[6]The use of $V(z,x)$ helps simplifying the problem by eliminating the need for carrying both x and $\widetilde{x}$ for the problem of agents. In particular, because of the use of $V(z,x)$, the dividend function $d(z,x)$ does not need $\widetilde{x}$ as an argument. Since $d(z,x)$ is small relative to other income or consumption of agents, using $V(z,x)$ instead of $\widetilde{V}(z,\widetilde{x})$ does not significantly affect the main result, although accuracy of the forecasting function $d(z,x)$ is negatively affected (See Table 8 in Appendix A).

hours worked, $\ell(z,x,s,a)$, the value of the firm matched with a worker of type (s,a) can be defined by the following Bellman equation:

$$J(z,x,s,a) = \max_{k\geq 0}\left\{ j(z,x,s,a) + (1-\lambda)\sum_{z'}\sum_{s'}\pi^z_{zz'}\pi^s_{ss'}\frac{J(z',x',s',a')}{1+r(z',x')}\right\} \tag{23}$$

subject to

$$j(z,x,s,a) = (e^z F(k, s\ell(z,x,s,a)) - (r(z,x)+\delta)k)(1-w(z,x,s,a)) \tag{24}$$

$$a' = g_a(z,x,1,s,a) \tag{25}$$

$$x' = f_x(z', \widetilde{f}_x(z,x)) \tag{26}$$

Note that the formulation above already takes into account that the value of being unmatched, $\widetilde{J}(z,\widetilde{x})$, is zero due to free entry (see equation (20)). $j(z,x,s,a)$ is the function for the current profit of a matched firm, which is the total surplus multiplied by the firm's share out of the surplus $(1-w(z,x,s,a))$. The total surplus is the output generated by the match minus the rental cost of capital.

Notice that the problem faced by the firm is virtually static; the current choice of k does not affect the future value of the firm. Therefore, it is easy to see that the optimal capital input $k(z,x,s,a)$ satisfies the following marginal condition:

$$r(z,x) = e^z F_k(k(z,x,s,a), s\ell(z,x,s,a)) - \delta \tag{27}$$

Moreover, applying Euler's theorem to the production function and the marginal condition just obtained above, a simple formula for the current profit can be obtained as follows:

$$\begin{aligned} j(z,x,s,a) &= e^z F_\ell(k(z,x,s,a), s\ell(z,x,s,a))(1-w(z,x,s,a)) \\ &= p(z,x)s\ell(z,x,s,a)(1-w(z,x,s,a)) \end{aligned} \tag{28}$$

Using Equation (28), the Bellman equation (23) can be simplified into the following:

$$J(z,x,s,a) = p(z,x)s\ell(z,x,s,a)(1-w(z,x,s,a)) + (1-\lambda)\sum_{z'}\sum_{s'}\pi^z_{zz'}\pi^s_{ss'}\frac{J(z',x',s',a')}{1+r(z',x')} \tag{29}$$

where $p(z,x)$ is determined by the usual marginal condition.

$$p(z,x) = e^z F_\ell(k(z,x,s,a), s\ell(z,x,s,a)) \tag{30}$$

4.11 Bargaining Outcome

The worker and the firm in a matched pair bargain jointly over the hours worked, $\ell(z,x,s,a)$, and surplus sharing rule, $w(z,x,s,a)$. I use the generalized Nash bargaining solution to determine the bargaining outcome. Let μ and $1-\mu$ denote the Nash bargaining parameter for the worker and the firm, respectively. The bargaining outcome $(\ell(z,x,s,a), w(z,x,s,a))$ can be formalized as follows:

$$(31)\quad (\ell(z,x,s,a), w(z,x,s,a)) = \underset{\ell,w}{\operatorname{argmax}} \left(W(z,x,1,s,a;\ell,w) - W(z,x,0,s,a)\right)^{\mu} J(z,x,s,a;\ell,w)^{1-\mu}$$

where $W(z,x,1,s,a;\ell,w)$ and $J(z,x,s,a;\ell,w)$ are the values of matched workers and firms conditional on the current (ℓ,w), respectively. $W(z,x,0,s,a)$ is the value of the worker conditional on being unemployed. Notice that it is already taken into account that the value of an unmatched firm (which is the reservation value of the bargaining firm) is zero in equilibrium because of free entry. Also notice that the value of an unemployed worker or the reservation value of the worker in the bargaining does not depend on (ℓ,w).

4.12 Equilibrium

Definition 1 (Recursive equilibrium) *A recursive equilibrium is a list of functions $W(z,x,e,s,a)$, $J(z,x,s,a)$, $\widetilde{J}(z,\widetilde{x})$, $g_a(z,x,e,s,a)$, $g_c(z,x,e,s,a)$, $\ell(z,x,s,a)$, $w(z,x,s,a)$, $r(z,x)$, $p(z,x)$, $d(z,x)$, $\widetilde{V}(z,\widetilde{x})$, $t(z,x)$, $f_x(z,\widetilde{x})$, and $\widetilde{f}_x(z,x)$ such that:*

1. *$W(z,x,e,s,a)$ is a solution to the worker's optimization problem, and $g_a(z,x,e,s,a)$, and $g_c(z,x,e,s,a)$ are associated optimal decision rules for saving and consumption, respectively.*
2. *$J(z,x,s,a)$ is a solution to the matched firm's optimization problem.*
3. *Free entry of firms, i.e., $\widetilde{J}(z,\widetilde{x}) = 0$. The number of vacancies $\widetilde{V}(z,\widetilde{x})$ is determined such that the free-entry condition is satisfied.*
4. *The law of motion associated with x, characterized by $f_x(z,\widetilde{x})$ and $\widetilde{f}_x(z,x)$, is consistent with the optimal decision rule for capital holding $g_a(z,x,e,s,a)$ and the law of motion associated with the individual productivity shock s and employment status e.*
5. *The rental price of capital and wage per efficiency unit of labor are determined by the following marginal conditions, with aggregate stock of capital (K) and labor (L) defined by (8) and (9), respectively.*

$$r(z,x) = e^z F_k(K(x), L(z,x)) - \delta$$

$$p(z,x) = e^z F_\ell(K(x), L(z,x))$$

6. *$(\ell(z,x,s,a), w(z,x,s,a))$ is determined as the solution to the generalized Nash bargaining between a worker and a firm.*
7. *$t(z,x)$ satisfies the following government budget constraint:*

$$\int_X t(z,x)dx = \int_X \mathbb{1}_{e=1} p(z,x) s\ell(z,x,s,a) w(z,x,s,a)\tau dx - \int_X \mathbb{1}_{e=0} b(s,a) dx$$

8. *The lump-sum dividend $d(z,x)$ is determined as the total profits of the current period net of the total costs of vacancy posting, as in (22).*

Table 2: Summary of the baseline calibration

Parameter	Value[1]	Remarks
β	0.9837	Time discount factor. Calibrated to match $K/Y = 10^1$.
σ	1.5000	Coefficient of relative risk aversion.
ψ	5.2210	Calibrated to match the average hours worked to be 0.33.
η	0.5000	Frisch labor supply elasticity.
θ	0.2890	Capital share of output.
δ	0.0150	Depreciation rate of capital.
ρ_z	0.9795	Persistence of TFP shock.
σ_z	0.0044	Standard deviation of innovation to TFP shock.
ρ_s	0.9956	Persistence of individual productivity shock.
σ_s	0.0323	Standard deviation of innovation to individual productivity shock.
α	0.6600	Matching elasticity. Calibrated to match relative size of σ_U and σ_V.
λ	0.1000	Separation probability.
γ	0.6246	Job finding probability.
κ	0.1148	Cost of posting a vacancy.
χ	0.6400	Replacement rate of unemployment insurance benefit.
τ	0.0370	Unemployment insurance tax rate.
μ	0.0701	Nash bargaining parameter for workers.

[1] Quarterly value.

5 Calibration

A model period is assumed to be a quarter. Table 2 summarizes the parameter values, with remarks about how each of the parameters is calibrated. The parameters can be classified into three groups based on how they are calibrated. The first group of parameters is calibrated based on independent empirical evidence (calibration does not require solving the model). The second group of parameters is calibrated using the steady-state version of the model, which is the baseline model described in the previous section but without the aggregate TFP shock. Each of the parameters are calibrated, jointly with other parameters, to match the time series average of the closely related data. The third set of parameters is calibrated using the model with aggregate shock. Each parameter is calibrated to match a cyclical statistic that is closely related to the parameter. More detailed descriptions of the calibration strategy follow.

5.1 Preferences

For the period utility function, I use the following functional form known as the Greenwood-Hercowitz-Huffman (GHH) preference, which is developed in Greenwood et al. (1988).

$$u(c,h) = \frac{\left(c - \psi \frac{(1-h)^{1+1/\eta}}{1+1/\eta}\right)^{1-\sigma}}{1-\sigma} \tag{32}$$

The unique feature of the GHH preference is that the preference implies there is no income effect. The GHH preference is extensively used in the small open economy models and recently in models with news shocks, since the preference helps these models to better replicate the data, mainly through a strong response of labor supply to productivity shocks. In the current model, the reason is different; with the standard utility function, the hours worked as a bargaining solution keep decreasing as the asset holding (and consumption) increases, and eventually the value of being unemployed exceeds the value of being employed, when the hours worked reach zero. This is because being unemployed entitles a worker to the unemployment insurance benefits with zero labor supply, while employed workers do not receive labor income or the benefits if labor supply is zero. On the other hand, since the optimal labor supply is independent of the asset holding with the GHH preference, it is possible to make sure that the value of being unemployed does not exceed the value of being employed as the asset holding increases. This property helps to avoid the problem that workers with a large amount of wealth find it optimal to stop searching and stay unemployed.[7]

The parameter representing the coefficient of relative risk aversion, σ, is set at 1.5, which is the standard value in the literature. η represents the Frisch elasticity of labor supply. Considering that the empirical estimates of η are between 0.1 and 1, I choose $\eta = 0.5$. The parameter ψ is calibrated endogenously (i.e., jointly with other parameters) such that, in the steady state of the model, the average hours worked is 0.33 of the disposable time of employed workers. This is broadly consistent with the evidence from the time-use survey. The time discount factor β is also calibrated endogenously. In particular, β is calibrated such that the annualized (quarterly) capital output ratio in the steady-state version of the model is 2.5 (10). I obtain $\beta = 0.9837$.

5.2 Production Technology

The production function takes the Cobb-Douglas form as follows:

(33) $$Y = e^z F(K, L) = e^z K^\theta L^{1-\theta}$$

θ is calibrated to match the average capital share of income in the U.S. economy computed using data from the National Income and Product Accounts (NIPA), which is 0.289. This number is slightly lower than the commonly used value, which is around one-third, because firms' profits are not included in the capital share. Instead, firms' profits are part of the total surplus, which is associated with the proportion $1 - \theta$. The depreciation rate δ is set at 0.015 (6.0 percent annual depreciation). The depreciation rate δ is calculated using the average ratio of total capital consumption over total capital stock (computed using NIPA data).

The TFP shock z is constructed by approximating the following AR(1) process using the method developed by Ada and Cooper (2003) with the number of abscissas $n_z = 5$.

(34) $$z' = \rho_z z + \epsilon_z$$

where $\epsilon_z \sim N(0, \sigma_z^2)$. Typically ρ_z is set at 0.95 and σ_z is set at 0.007 based on the estimates obtained by Cooley and Prescott (1995) using the Solow residuals. However, when the original AR(1) process is approximated using a Markov chain with a relatively small number of abscissas, there is no guarantee that the obtained Markov process replicates important characteristics of

[7] Although it can be an interesting model of wealth-rich workers withdrawing from the labor market or retiring early, this is not the focus of the current paper.

the original AR(1) process. Therefore, I calibrate ρ_z and σ_z such that the obtained Markov chain replicates the unconditional variance and autocorrelation of the AR(1) process with the persistence of 0.95 and the standard deviation of 0.007. This process generates $\rho_z = 0.9795$ and $\sigma_z = 0.0044$.

The individual productivity shock is constructed in the same way as z; the following AR(1) process is approximated using the method developed by Ada and Cooper (2003) with the number of abscissas $n_s = 3$.

(35) $s' = \rho_s s + \epsilon_s$

where $\epsilon_s \sim N(0, \sigma_s^2)$. There are various estimates of the autocorrelation and the standard deviation of labor income. According to Domeij and Heathcote (2004), existing studies estimate the autocorrelation of labor income to lie between 0.88 and 0.96 and the standard deviation of labor income in the range of 0.12 and 0.25. Therefore, I calibrate ρ_s and σ_s such that when approximated by a Markov chain, the obtained Markov chain exhibits the unconditional autocorrelation of 0.90 and the unconditional standard deviation of 0.20. Notice that it is not a trivial task because the targets are associated with labor income $s\ell$, which includes an endogenous variable ℓ, and not s alone. In order to calibrate the parameters for the individual productivity shocks, the model must be solved repeatedly until the targets are satisfied. The process yields $\rho_s = 0.9956$ and $\sigma_s = 0.0323$.

5.3 Job Turnover Technology

Following Shimer (2005), the following Cobb-Douglas function is used for the aggregate matching function:

(36) $f_m(S, V) = \gamma S^{\alpha} V^{1-\alpha}$

The estimates for α are wide-ranging.[8] Shimer (2005) uses the unemployment data of the Bureau of Labor Statistics (BLS) and the help-wanted advertising index constructed by the Conference Board and obtains $\alpha = 0.72$. Hall (2005) uses Job Openings and Labor Turnover Survey (JOLTS) data on vacancies, unemployment, and job-finding probability and obtains $\alpha = 0.235$. Blanchard and Diamond (1989) use Current Population Survey (CPS) data to construct the data on unemployment and new matches and the help-wanted advertising index of the Conference Board and obtain $\alpha = 0.4$. In order to calibrate α, I use the property that α affects the relative size of the volatility of unemployment and vacancies. Remember that the volatility of vacancies (14.1 in the U.S. data) is roughly close to that of unemployment (13.0). I calibrate α to match the relative size of the volatilities of the model. The procedure pins down α to be 0.66, which is close to the middle of the various estimates.

The separation probability λ is set at 0.10, which makes the average tenure of a job in the model consistent with the average tenure in the data (2.5 years, or 10 quarters). Since a worker whose match just got destroyed can immediately start searching for the next job, the job finding probability f_w and the separation probability λ have to satisfy the following relationship in the steady state:

(37) $f_w(U + (1 - U)\lambda) = (1 - U)\lambda$

[8] See Petrongolo and Pissarides (2001) for summary of empirical estimates.

With $\lambda = 0.10$ and $U = 0.0567$ (the U.S. average unemployment rate), equation (37) implies that the job finding probability f_w is 0.6246. The average V/S ratio is normalized to be one. This normalization plus the average job finding probability of 0.6246 yields $\gamma = 0.6246$. The parameter for the cost of posting a vacancy κ is calibrated such that in the steady state of the model, V/S is actually one. Using this procedure, $\kappa = 0.1148$ is obtained for the baseline model.

5.4 Market Structure

The lower bound of the capital stock holding, $\underline{a}$, is set at zero in the baseline model. In one of the experiments implemented in Section 7, I will investigate how a relaxation of the borrowing limit affects the cyclical properties of the model.

5.5 Unemployment Insurance

For the standard labor search and matching model, it is not trivial to agree on the target value of the replacement rate for unemployment insurance benefits for two reasons. First, a variety of numbers could be justified as the monetary value of being unemployed, mainly because the wage (before becoming unemployed) distribution of unemployed workers is typically very different from the wage distribution of the entire labor force. Second, even if the monetary value of being unemployed is pinned down, if the model does not explicitly include leisure (or more generally, non-monetary benefits of unemployment), the value of additional leisure when a worker is unemployed needs to be imputed and the replacement rate needs to be adjusted.

As for the first point, the unemployment insurance benefit replaces around 60 percent of past earnings in the U.S. (Hornstein et al. (2005)). However, since those who are unemployed tend to earn less than the average worker, the ratio of the average unemployment insurance benefit and average earnings is substantially below 60 percent. According to the BLS, the ratio of the average weekly benefit and the average weekly wage is 35 percent between 1951 and 2004. Hornstein et al. (2005) suggest the value of 20 percent.

As for the second point, Shimer (2005) uses 40 percent as the value of being unemployed relative to being employed, including the imputed value of leisure. Alternatively, Hagedorn and Manovskii (2008a) do not set the imputed value of leisure a priori but instead calibrate the replacement rate to match the fact that in response to a 1 percent increase in labor productivity, the wage increases by 0.45 percent. As a result, the replacement rate is calibrated to be 95.5 percent. At least a part of the difference between 40 percent and 95.5 percent is attributed to the value of leisure, and other extra utility in unemployment. As pointed out clearly in Hornstein et al. (2005), the difference in the calibrated value of the replacement rate between Shimer (2005) and Hagedorn and Manovskii (2008a) generates the difference in the model's ability to amplify productivity shocks.

For the calibration of the baseline model, I follow the strategy employed by Hagedorn and Manovskii (2008a); I do not set any prior with respect to the instantaneous utility of being unemployed and calibrate the replacement rate parameter χ such that the baseline model generates the elasticity of the average labor income (measured as compensation per worker) relative to labor productivity (measured as output per worker) to be 0.56, which is the empirical value obtained from the data used for Table 1. This procedure produces $\chi = 0.640$, which generates the elasticity of 0.56. Notice that the calibrated χ is surprisingly close to 0.60, which is the replacement rate relative to the after-tax income during the first six months after losing a job, according to the calculation of the Organization for Economic Cooperation and Development (OECD) (1996) and quoted by Hornstein et al. (2005). Notice that the second issue in calibrating the replacement rate parameter, which is raised in the preceding paragraph, is not an issue here because the value of leisure is explicitly modeled in the current model. The value of leisure is indirectly pinned down when the parameter ψ is calibrated such that average hours worked in the model are close to the average hours worked in the U.S. economy. Notice that there is no freedom in choosing the value of leisure. In other words, the comparison between the parameter χ and the replacement rate of the unemployment benefits are more direct than the existing studies.

Besides, the 4 percent difference between the calibrated value of χ (64 percent) and the actual replacement rate of unemployment insurance benefits of 60 percent can probably be reconciled by elements omitted in the model. For example, on the employment side, commuting costs, training costs, and child-care costs while working would effectively reduce the income from employment net of all costs. On the unemployment side, if elements such as irregular jobs available during unemployment spells or insurance mechanism absent from the current set-up (for example, staying at parents' house while unemployed) are taken into account, a replacement rate of 64 percent might well be reasonable. Overall, it is reasonable to argue that obtaining the value of unemployment insurance benefits of 64 percent through the baseline calibration procedure to be a success.

The unemployment insurance benefits $b(s,a)$ are set such that, in the steady state, the benefit received by a worker of type (s,a) is exactly the proportion χ of the disposable labor income that the worker would receive if the worker were employed. Formally, $b(s,a)$ is set as follows:

$$b(s,a) = \chi \overline{p} s \overline{\ell}(s,a) \overline{w}(s,a)(1-\tau) \tag{38}$$

where $\overline{p}$, $\overline{\ell}(s,a)$, and $\overline{w}(s,a)$ are the labor productivity, hours worked by a type (s,a) worker, and surplus sharing rule of a match associated with a type (s,a) worker in the steady state, respectively.

Finally, τ is calibrated such that the government budget balances in the steady state. In other words, τ is calibrated such that $\overline{t} = 0$ in the steady state. This calibration strategy yields $\tau = 0.0370$ in the baseline calibration.

5.6 Bargaining

The Nash bargaining parameter μ is calibrated such that the average shares of workers and firms out of the total surplus in the steady-state version of the model match the empirical counterparts. According to NIPA, after-tax corporate profits are about 3.3 percent of the sum of the after-tax corporate profits and labor income, which corresponds to the firms' share out of the total surplus in the model. Neither Shimer (2005) nor Hagedorn and Manovskii (2008a) targets the size of the shares of workers and firms, but their calibration strategies imply that the average share of firms out of the total surplus is close to 3 percent. This calibration strategy yields $\mu = 0.0701$ in the baseline model.

6 Computation

6.1 Approximate Equilibrium

To compute an equilibrium of the model, I use the solution method developed by Krusell and Smith (1998); I focus on the stationary stochastic recursive equilibrium and compute the approximation of the true equilibrium. The key component of Krusell and Smith's (1998) approximation method is to use a finite set of statistics of the type distribution x to represent x. As a result, instead of dealing with an infinitely dimensional object x, one only needs to deal with a finite set of statistics of x. An interpretation of this approximation method is that agents in the model are allowed to use partial information of the entire type distribution x when they make decisions. In this sense, the approximate equilibrium can be called the *equilibrium with partial information* or the *bounded rationality equilibrium*. The important finding of Krusell and Smith (1998) is that, in their model with both aggregate and uninsured idiosyncratic shocks, using a very small set of statistics that represent the type distribution is sufficient to achieve a *good* approximation, in the sense that using more statistics of x does not change the properties of the model in a sizable manner. This insight helps to greatly reduce the computational cost in the current model as it does for theirs.

Following the finding of Krusell and Smith (1998), I use the first moment of the asset distribution K, and the measure of the employed N to represent the type distribution x. The model here is more challenging than the model solved by Krusell and Smith (1998), for two reasons. First, since both the aggregate capital stock and the total measure of employed workers (or the total measure of unemployed workers) must be a part of the aggregate state variables, the number of aggregate state variables must be at least two, instead of one in Krusell and Smith (1998). Second, it is necessary to simultaneously find five, instead of one, forecasting functions that are consistent with the optimal decisions of workers and firms.

6.2 Computation of the Approximate Equilibrium

Computing the approximate equilibrium involves finding five equilibrium forecasting functions, which map the aggregate state variables (z, K, N) into capital stock in the next period, K', capital output ratio in the current period, K/Y, number of vacancies posted, V, amount of dividends, d, and government transfer, t, respectively. The five functions are taken as given in the optimization problems of workers and firms and must be consistent with the resulting optimal decisions. Following Krusell and Smith (1998), the five forecasting functions are parameterized and thus characterized by a set of coefficients. Finding an equilibrium is equivalent to finding a set of coefficients with which the parameterized forecasting functions are consistent with the aggregate dynamics implied by the optimal decisions of workers and firms.

The value function is approximated using a spline with respect to individual capital holding a. The optimal decision rules for future capital holding, consumption, and hours worked are approximated using piecewise-linear functions with respect to current capital holding. Both value functions and the optimal decision rules are interpolated with piecewise-linear functions with respect to the aggregate state variables (K, N). The problem of the workers as well as that of firms is solved using the value function iteration. Details of the computational methods, including a detailed discussion of how to construct the approximate equilibrium, are found in

Appendix A.

7 Results

The first section looks at the aggregate properties of the baseline model economy in the steady state. Section 7.2 presents the main result of the paper – the cyclical properties of unemployment and vacancies. Comparison of performance with related models is also provided. Section 7.3 provides analysis of the main result. Section 7.4 presents the other main result, which is the business cycle properties of the baseline model economy.

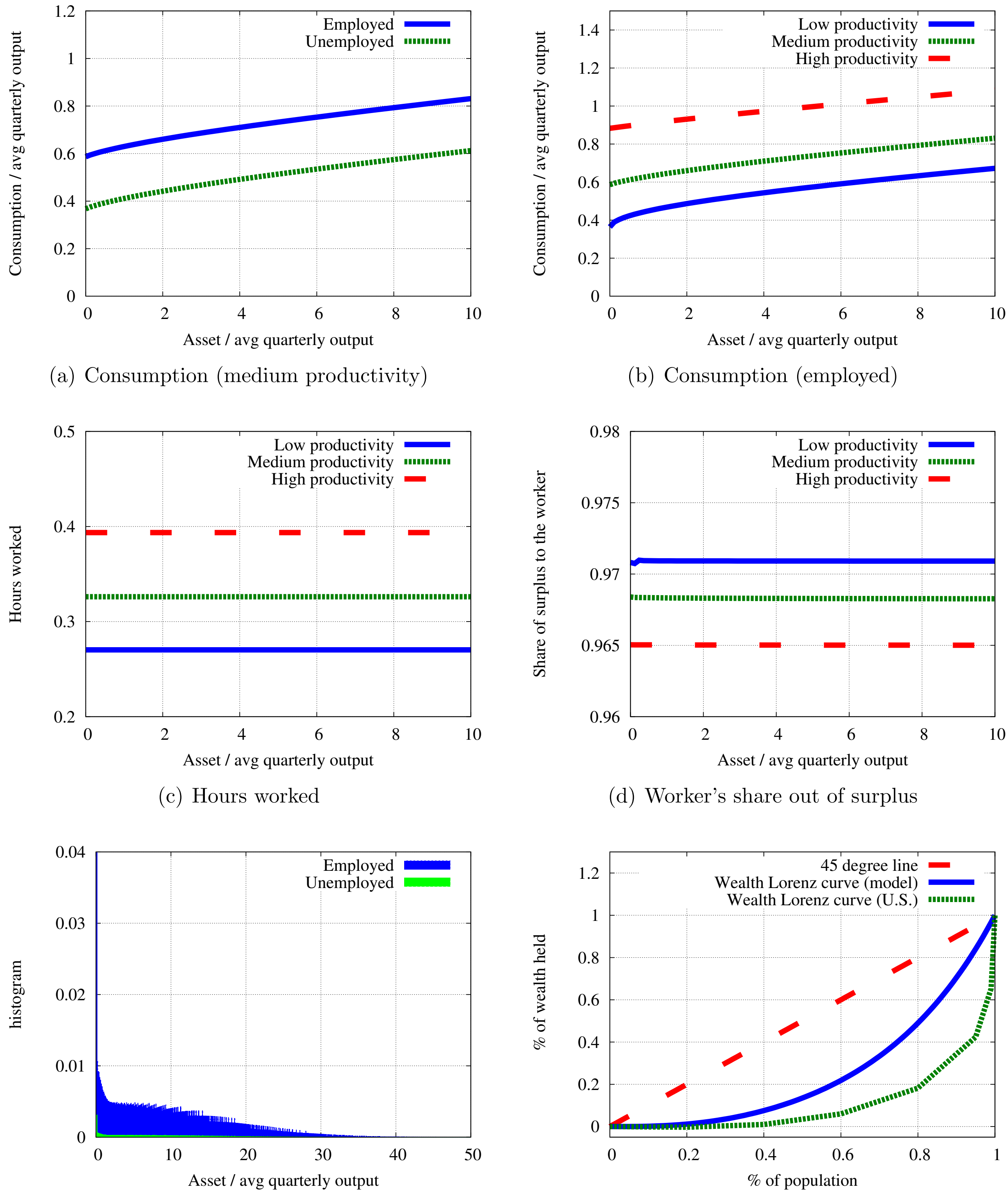
7.1 Aggregate Properties in the Steady State

Figure 1 illustrates properties of the baseline model economy in the steady state from various dimensions. First, Figure 1(a) compares the optimal consumption schedule for employed and unemployed workers with the medium productivity. The figure shows that employed workers consume more than unemployed workers of the same type (productivity and asset holding). The size of the difference in consumption is 26 percent for a worker with medium productivity and mean asset level. The size of the consumption drop is not far from some of the empirical estimates. For example, Burgess et al. (1981) find that the average household reduced its expenditures by 15.2 percent, but 40 percent of households reduced their expenditures by 20 percent or more, from the month prior to job loss to the 13th week of unemployment. On the other hand, Gruber (1997) finds that the average size of the consumption drop upon job loss is 6.8 percent, although he uses annual food consumption expenditures data from the Panel Study of Income Dynamics (PSID); it is likely that the drop in food consumption upon losing a job is smaller than the drop in total consumption.

Figure 1(b) compares the optimal consumption schedules for employed workers with different levels of productivity. Not surprisingly, the higher the productivity is, the more they consume, conditional on the asset holding. There is a substantial curvature for low-productivity workers with small levels of assets because their optimal consumption decision is constrained.

Figure 1(c) and Figure 1(d) show the bargaining outcome for workers with different productivity and asset holding. Let's start from hours worked (Figure 1(c)). Since the GHH preference is used, hours worked do not depend on consumption or asset holding. As expected, workers with higher productivity work longer hours. Interestingly, Figure 1(d) shows the inverse order; workers with higher productivity receive a smaller share out of the total surplus. Dispersion of the labor supply measured in efficiency units is larger than the dispersion of labor productivity, because more productive workers work longer hours, but the dispersion of labor income is smaller than the dispersion of the labor supply, because the share out of the total surplus associated with the more productive workers is smaller. In other words, the Nash bargaining solution compresses the dispersion of labor income. It is a natural consequence of a bargaining outcome between a risk-averse worker and a risk-neutral firm. The risk-neutral firm is giving insurance to the risk-averse worker by giving more out of the total surplus when productivity is lower, and the opposite when productivity is higher.

Figure 1(e) and Figure 1(f) exhibit the distribution of asset holdings in the steady state. Figure 1(e) shows that there is a spike at the bottom of the asset distribution. Figure 1(f) compares the Lorenz curve for the baseline model as well as the U.S. economy. The Lorenz



(a) Consumption (medium productivity)

(b) Consumption (employed)

(c) Hours worked

(d) Worker's share out of surplus

(e) Histogram of asset distribution

(f) Lorenz curve of asset distribution

Figure 1: Aggregate properties of the baseline model economy

Table 3: Cyclical properties of the baseline model economy[1]

Variable	SD%	Relative SD%[2]	Auto-corr	Cross-correlation of output with x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}
Output	1.51	1.00	0.73	0.48	0.73	1.00	0.73	0.48
Consumption	0.76	0.50	0.74	0.44	0.70	0.99	0.75	0.54
Investment	5.22	3.45	0.72	0.51	0.74	0.99	0.70	0.44
Unemployment rate (u)	10.91	7.21	0.81	−0.44	−0.69	−0.97	−0.82	−0.59
Vacancies (v)	10.86	7.18	0.53	0.50	0.71	0.93	0.48	0.24
Market tightness (v/u)	18.40	12.16	0.83	0.43	0.68	0.96	0.86	0.62
Job finding probability	4.66	3.08	0.67	0.49	0.72	0.97	0.65	0.41
Aggregate hours	0.92	0.61	0.78	0.47	0.72	0.99	0.77	0.54
Employment	0.65	0.43	0.83	0.44	0.68	0.94	0.80	0.58
Average weekly hours	0.31	0.20	0.65	0.48	0.71	0.97	0.63	0.38
Labor share	0.41	0.27	0.64	−0.48	−0.71	−0.97	−0.61	−0.36
Output per hour	0.62	0.41	0.65	0.48	0.71	0.97	0.63	0.38
Wage	0.21	0.14	0.69	0.47	0.70	0.96	0.65	0.42
Output per worker	0.93	0.61	0.65	0.48	0.71	0.97	0.63	0.38
Compensation per worker	0.52	0.34	0.67	0.48	0.71	0.97	0.64	0.40

[1] All data are in logs and filtered using the H-P filter with a smoothing parameter of 1600.

[2] Relative to the standard deviation of output.

curve from the U.S. economy is based on the findings of Budría et al. (2002). Most important but not surprising, the model cannot generate the high concentration of assets observed in the U.S. economy, because several elements are missing from the model. First, although the stochastic process for individual labor productivity is calibrated to match the empirical dispersion of earnings, the empirical dispersion of earnings is typically computed from a data set that under-samples extremely rich households, many of whom are entrepreneurs. Second, the model abstracts from life-cycles, especially the young with few assets. Third, job loss and job finding probabilities are assumed to be the same across all workers in the model for simplicity, but job finding probability tends to be higher for higher income workers in data. However, I found that the cyclical properties of the model do not change substantially even if the model is calibrated to have lower aggregate savings so that more workers are close to the borrowing limit.

7.2 Cyclical Properties of Unemployment and Vacancies

Table 3 summarizes the business cycle properties of the baseline model economy. The table is the exact counterpart of Table 1, which exhibits the business cycle properties of the post-war

U.S. economy. In this section, the cyclical properties of unemployment, vacancies, and related variables are examined, since they are of primary interest in this paper. The next section offers analysis. In Section 7.4, the cyclical properties of the other variables are investigated.

The most important success of the baseline model is that the model economy replicates very high volatility of unemployment and vacancies, which Shimer (2005) claims the standard Mortensen-Pissarides model cannot replicate. The standard deviation of the detrended unemployment rate in the model economy is 10.91 percent, which is substantially higher than that of output (1.51 percent) and close to the U.S. economy counterpart (12.95 percent). In terms of the unemployment rate volatility relative to output volatility, the model generates 7.21, while it is 8.19 in the data. For a comparison, Shimer (2005) finds that in the standard Mortensen-Pissarides model with productivity shocks, the standard deviation of unemployment is 0.9 percent, making the relative volatility of unemployment 0.45 percent.[9] It is worth emphasizing that the current model economy generates a high volatility of unemployment, while the replacement rate of the unemployment insurance benefits (0.64) turned out to be a level comparable to the one in the U.S. economy.

The baseline model also captures other cyclical properties of unemployment. Autocorrelation in the model is 0.81, which is strong, albeit slightly lower than its U.S. economy counterpart (0.87). Unemployment is strongly countercyclical in both economies; the contemporaneous correlation between unemployment and output is -0.84 in the U.S. and -0.97 in the model. On the other hand, while unemployment lags the cycle in the U.S. economy, there is no lag in the baseline model, although the model generates a high persistence of unemployment. The correlation between output and unemployment in the next quarter in the U.S. economy (-0.85) is close to the same statistics of the baseline model (-0.82).

The baseline model also succeeds in replicating a high observed volatility in the number of vacancies. The standard deviation of the number of vacancies in the model is 10.86 percent. It is also significantly higher than the standard deviation of output (1.51 percent), as in the U.S. data, although the absolute level is slightly lower than the level in the U.S. economy (14.10 percent). Volatility of vacancies relative to the output volatility is 8.91 in the U.S. data and 7.18 in the baseline model. For a comparison, the Mortensen-Pissarides model calibrated by Shimer (2005) yields relative volatility of 1.35.[10] Both in the U.S. and in the model economies, vacancies are strongly procyclical and do not lead or lag the cycle; the contemporaneous correlation with output is 0.89 in the U.S. economy, while it is 0.93 in the model economy. However, vacancies in the model are not as persistent as in the data, which causes the lack of the lag in unemployment. The autocorrelation of the number of vacancies posted is 0.91 in the data, while it is 0.53 in the model economy. The correlation between output and lagged vacancies is 0.82 in the data but 0.48 in the model.

The market tightness (the ratio between the number of vacancies and the unemployment rate) is substantially more volatile than output, and strongly procyclical, in both the model and

[9]Note that Shimer (2005) uses labor productivity as the basis of comparison and uses a different parameter (10^5 instead of 1600) for the H-P filter. The standard deviation of H-P-filtered labor productivity and unemployment is 2.0 percent according to his calculation. But the basic message that the Mortensen-Pissarides model does not exhibit strong amplification is not affected by these differences.

[10]See footnote 9.

Table 4: Comparison of cyclical properties of unemployment and vacancies[1]

Economy	σ_U	σ_V	$\sigma_{V/U}$	$\rho(U_t, V_t)$	$\rho(U_{t-1}, V_t)$
U.S.	12.95	14.10	26.44	−0.91	−0.91
Baseline model	10.91	10.86	18.40	−0.85	−0.77
Andolfatto (1996)	1.05	4.58	4.86	−0.19	−0.65
Merz (1995)	4.95	6.83	n.a.	−0.15	−0.82

[1] All data are quarterly, in logs, and filtered using the H-P filter with a smoothing parameter of 1600. σ and ρ denote standard deviation and correlation, respectively.

the U.S. economies. The volatility of the market tightness relative to output volatility is 12.16 in the model, while it is 16.71 in the data. The contemporaneous correlation with output is 0.96 in the baseline model and 0.89 in the data. Job finding probability in the model is also much more volatile than output and strongly procyclical, as in the data. Its standard deviation is 4.66 percent in the model and 8.38 percent in the U.S. economy. The contemporaneous correlation between output and the job finding probability is higher in the model (0.97) than in the U.S. economy (0.82).

7.3 Analysis

7.3.1 Comparison with Previous Models with Labor Market Frictions

Although real business cycle models with labor market search and matching (Andolfatto (1996) and Merz (1995)) have similar success in terms of replicating various dimensions of the business cycle properties of the U.S. economy, as shown in the next section, the current model is better in achieving these while generating the observed cyclical properties of unemployment and vacancies. Table 4 summarizes the comparison. In the models by Andolfatto (1996) and Merz (1995), volatilities of unemployment and vacancies are substantially lower compared with those of the U.S. economy, while the current model economy exhibits a substantially higher variation in unemployment and vacancies.[11]

Moreover, regarding the strong negative correlation between the two (usually expressed as the downward sloping Beveridge curve), the baseline model generates a high negative correlation between unemployment and vacancies, which is close to the empirical counterpart, while the previous two models generate a far weaker negative correlation. It is shown in the fourth column of Table 4. The correlation is −0.91 in the data and −0.85 in the baseline model, while it is −0.19 in Andolfatto (1996) and −0.15 in Merz (1995). The failure of the models by Andolfatto (1996) and Merz (1995) is partly due to the timing assumption; both assume that vacancies are posted after production takes place and consequently unemployment adjusts in the next period, which

[11] Andolfatto (1996) relies on the assumption that the mean unemployment rate is 43 percent; it is assumed that all of the people out of the labor force are actually searching for a job as well. Since the proportion of the unemployed in his definition is large, the number of vacancies becomes responsive to shocks. That is why log of the number of vacancies exhibits a decent level of volatility. On the other hand, the volatility of log-unemployment rate is very small, because the size of the average unemployment rate is very high.

implies that a lag of at least a quarter (one period in the model) between aggregate shocks to output and adjustments in the labor market is forced. On the other hand, in the current model, vacancies are posted and unemployment adjusts before production occurs; unemployment is not forced to adjust one quarter after the number of vacancies adjusts. To make the point clearer, let us look at the correlation between vacancies and lagged unemployment. It is shown in the last column. The negative correlation is stronger and closer to the data (-0.91) in the models by Andolfatto (1996) (-0.65) and Merz (1995) (-0.82), although the baseline model generates a fairly strong negative correlation (-0.77) as well.

In sum, the baseline model successfully generates a high volatility of the unemployment rate and number of vacancies posted and a strong negative correlation between the two, while previous models failed to generate these features. While the previous models succeed in generating the strong persistence of unemployment and consequently the lag of unemployment over the business cycles, this feature is generated by the assumption that unemployment adjusts with a period of delay. Even without such an assumption, the baseline model generates a relatively strong persistence of unemployment. I expect that the baseline model would be improved by having a stronger persistence if vacancies are modeled as stock instead of flow, as in Fujita and Ramey (2007). They generate a sluggish and hump-shaped response of the number of vacancies within the standard Mortensen-Pissarides model. I will leave this extension for future research.

7.3.2 Calibration Strategy

Why can the baseline model generate strong amplification? Since the intuition is closely related to the result by Hagedorn and Manovskii (2008a), I will first go over their calibration strategy, and then compare the calibration strategy adopted for the baseline model with theirs in order to provide the answer to the question.

There are three important parameters in the calibration of Hagedorn and Manovskii (2008a): cost of posting a vacancy κ, worker's Nash bargaining share μ, and the parameter representing the flow utility of unemployment χ. In calibrating the three parameters, they first carefully compute the vacancy posting cost by looking at time spent on hiring and find that κ is small – 3 to 4.5 percent of the quarterly wages of a new hire. Through the free-entry condition, a small vacancy posting cost implies small profits for a firm from a match. Given the (small) size of the firms' profits, they calibrate μ and χ such that (i) the average labor market tightness that is consistent with the average unemployment rate, and (ii) the elasticity of wages with respect to productivity of 0.449, are satisfied by the model. Their calibration procedure yields a relatively small $\mu = 0.052$ and a very high replacement rate of $\chi = 0.955$. Mortensen-Pissarides model calibrated following their strategy turns out to have a strong amplification because small average profits of firms and real wage stickiness imply a large volatility of firms' profits. If the profits of the firms are volatile, so are the number of vacancies and accordingly unemployment.

Real wage stickiness can be obtained in their calibration because, with a high replacement rate, workers are close to indifferent between being employed and being unemployed. This small gain from being employed rather than being unemployed is consistent with a small bargaining power for a worker within a match (small value of μ). From this intuition, it is easy to see that a high replacement rate (Hagedorn and Manovskii (2008a) obtain 0.955) is an integral part of their calibration. Their result is consistent with the claim by Shimer (2005) and Hall (2005) that

Table 5: Cyclical properties of the model without labor-leisure choice[1]

Economy	σ_U/σ_Y	σ_V/σ_Y	$\sigma_{V/U}/\sigma_Y$	$\rho(U,V)$	σ_L/σ_Y[2]
U.S.	8.19	8.91	16.71	−0.91	1.00
Baseline model	7.21	7.18	12.16	−0.85	0.61
Model without leisure, $\chi = 0.400$	1.22	1.37	2.21	−0.88	0.07
Model without leisure, $\chi = 0.640$	1.87	2.11	3.41	−0.88	0.11
Model without leisure, $\chi = 0.955$	6.61	7.41	11.91	−0.86	0.41

[1] All data are quarterly, in logs, and filtered using the H-P filter with a smoothing parameter of 1600. χ represents the replacement rate of unemployment insurance benefits. σ and ρ denote standard deviation and correlation, respectively.

[2] L denotes total hours.

real wage stickiness can generate a stronger amplification of the Mortensen-Pissarides model.

Turning to the calibration strategy of the baseline model, first, instead of calibrating κ directly, I use the firms' aggregate profits over output as a target and calibrate κ to match the target. The size of the firms' profits is small (3.3 percent of GDP), which is consistent with the calibration of Hagedorn and Manovskii (2008a). I then employed the same calibration strategy to pin down $\mu = 0.0701$ and $\chi = 0.6400$. The baseline model developed in the current paper exhibits a strong amplification because of the relatively small μ, as in Hagedorn and Manovskii (2008a).

However, it is easy to notice that the replacement rate parameter χ is very different between the value of Hagedorn and Manovskii (2008a) (0.955) and the value for the baseline model (0.64). In Hagedorn and Manovskii (2008a), since the utility function is linear, average labor productivity is normalized to one, and there is no utility from leisure, χ represents the period utility from unemployment relative to that of employment. Naturally, although it is impossible to systematically distinguish, $\chi = 0.955$ is considered to include utility from unemployment other than from unemployment insurance benefits, in particular, more time for leisure. On the other hand, the baseline model developed here explicitly distinguishes utility from leisure and utility from unemployment insurance benefits, and χ captures only the latter. In other words, the calibration of the baseline model implies that a replacement rate of approximately 30 percent, which is the difference of χ between theirs and the baseline model, can be interpreted as representing the utility from leisure. The next section investigates the role of leisure further.

7.3.3 Role of Leisure

In order to further investigate the role of leisure in the baseline model, I compute the cyclical properties of the models without labor-leisure choice. Apart from the additional assumption that hours worked are inelastic and set at the average hours worked in the baseline model, the same calibration strategy is used whenever feasible, except for the replacement rate of unemployment insurance benefits (χ). I investigate three different values of χ: The first model has the replacement rate used by Shimer (2005) ($\chi = 0.400$). The second model shares the same replacement rate as the baseline model ($\chi = 0.640$). The comparison between this model and the baseline model helps us understanding the role of leisure. In the third model, I use the same replacement rate used in Hagedorn and Manovskii (2008a) ($\chi = 0.955$). Table 5 summarizes the cyclical properties of unemployment and vacancies in the models without leisure, with three different values of χ. The cyclical properties of the baseline model and the U.S. economy are included in the first part of the table, for comparison.

Five remarks are worth making. First, if the utility from leisure is turned off from the baseline model economy (see the model without leisure and replacement rate of 0.640), the strong amplification of the baseline model disappears. The model with a realistic value of χ can exhibit a strong amplification only with the utility from leisure. Second, the model without leisure but with the same replacement rate as Hagedorn and Manovskii (2008a) (0.955) exhibits strong amplification comparable to that in the baseline model. This lack of amplification in the model with $\chi = 0.400$ (same value as in Shimer (2005)) and the strong amplification in the model with $\chi = 0.955$ (value obtained by Hagedorn and Manovskii (2008a)) together imply that other features of the baseline model such as risk-averse preferences, neoclassical production technology, and incomplete markets do not significantly change the nature of the model from the simple Mortensen-Pissarides model analyzed by Shimer (2005) and Hagedorn and Manovskii (2008a). Third, the strength of amplification of the model without leisure monotonically increases with the replacement rate. Fourth, the correlation between unemployment and vacancies is similar across different models. Fifth, the relative (to output) volatility of total hours also increases with the replacement rate, but the relative volatility of total hours in the model without leisure cannot match that of the baseline (0.61) even with the replacement rate of 0.955 (0.41). This is because the models investigated here lack the intensive margin of labor supply adjustments. The intensive margin of labor supply adjustments also helps amplifying the initial shocks, although even the baseline model with the intensive margin cannot generate the volatility of total hours as large as in data.

In sum, the gains offered by the baseline model developed in the current paper are two-fold. First, the utility from leisure enables the model to generate a strong amplification with a reasonable level of the replacement rate of 0.64. Second, the intensive margin of labor supply adjustments increases the size of the fluctuations in the model and, in particular, makes the cyclical properties of hours closer to the data.

7.3.4 Role of Market Incompleteness

One of the novel features of the baseline model developed in the current paper is market incompleteness. How important is this feature in shaping the cyclical properties of unemployment and vacancies? To answer the question, three alternative models are investigated. First is the model with a relaxed borrowing constraint. The other two are models with different degrees of individual productivity shocks. Table 6 compares the cyclical properties of unemployment and vacancies of these alternative models (the bottom three rows) with those of the U.S. economy (first row) and the baseline model (second row).

As for the model with a relaxed borrowing constraint, the borrowing limit is relaxed to 12 months (4 quarters) of the average income in the model. Remember that the borrowing limit

Table 6: Cyclical properties of alternative models[1]

Economy	σ_U/σ_Y	σ_V/σ_Y	$\sigma_{V/U}/\sigma_Y$	$\rho(U,V)$
U.S.	8.19	8.91	16.71	−0.91
Baseline model	7.21	7.18	12.16	−0.85
Model with relaxed borrowing limit[2]	7.21	7.11	12.11	−0.85
Model without individual productivity shock	7.11	7.53	12.45	−0.86
Model with higher individual productivity volatility[3]	7.73	7.39	12.79	−0.85

[1] All data are quarterly, in logs, and filtered using the H-P filter with a smoothing parameter of 1600. σ and ρ denote standard deviation and correlation, respectively.

[2] Borrowing limit is set at annual (4 quarters) average income in the model.

[3] Standard deviation of individual productivity shocks in increased by 37 percent from the baseline.

is set at zero in the baseline model. Clearly from Table 6, there is no significant difference between the two models with respect to the cyclical properties of unemployment and vacancies. Why? Since workers in the baseline model are already well insured only by their own savings, a relaxed borrowing constraint does not significantly affect the choice of workers and thus the aggregate properties of the model. This is consistent with the well-established result that cyclical properties of macroeconomic aggregates are similar between the complete-markets real business cycle models and their incomplete-markets counterparts, as shown in Krusell and Smith (1998).

Regarding the role of the individual productivity shocks, two cases are investigated. First, I study the model without individual productivity shocks (fourth row of Table 6). All workers are given the average individual productivity of the baseline model, but they are still subject to aggregate TFP shocks and unemployment shocks. Second, I increase the standard deviation of the individual productivity shocks by 37 percent. According to Heathcote et al. (2010), the cross-sectional variance of log individual wages steadily increased from about 0.25 in the early 1970s to about 0.47 by 2005.[12] The 37 percent increase in the standard deviation of the individual productivity shocks corresponds to such change in the cross-sectional variance of wages.[13] The results shown in Table 6 are clear; again, the strength of the individual productivity shocks do not affect significantly the cyclical properties of unemployment and vacancies.

7.4 Business Cycle Properties

In this section, I will evaluate other cyclical properties of the baseline model (Table 3) by comparing them with those of the U.S. economy (Table 1). I also compare the baseline model developed in the current paper with the standard RBC models. Table 7 summarizes the cyclical properties

[12] See Figure 2(A) of Heathcote et al. (2010).

[13] In particular, I assume that the increase in the cross-sectional variance of wages was induced by an increase in the volatility of the individual productivity shocks and calibrate the standard deviation of individual productivity shocks to match the increase in the variance of wages.

Table 7: **Cyclical properties of real business cycle models**[1]

Variable	SD%	Relative SD%[2]	Auto-corr	Cross-correlation of output with x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}
Real business cycle (RBC) model with GHH preference								
Output	1.19	1.00	0.72	0.47	0.72	1.00	0.72	0.47
Consumption	0.59	0.50	0.73	0.41	0.68	0.99	0.75	0.53
Investment	4.68	3.93	0.71	0.50	0.73	0.99	0.68	0.42
Aggregate hours	0.40	0.33	0.72	0.47	0.72	1.00	0.72	0.47
Output per hour	0.79	0.67	0.71	0.47	0.71	1.00	0.71	0.47
Andolfatto (1996)[3]								
Output	1.51	1.00	0.81	0.58	0.81	1.00	0.81	0.58
Consumption	0.43	0.28	0.82	0.34	0.60	0.83	0.74	0.64
Investment	4.72	3.13	0.82	0.61	0.82	0.99	0.79	0.54
Aggregate hours	0.97	0.64	0.89	0.57	0.78	0.96	0.90	0.68
Employment	0.87	0.58	0.86	0.41	0.61	0.82	0.98	0.83
Average weekly hours	0.36	0.24	0.39	0.55	0.63	0.62	0.07	−0.17
Labor share	0.16	0.11	0.37	−0.50	−0.58	−0.58	−0.03	0.18
Output per hour	0.64	0.42	0.57	0.52	0.74	0.90	0.54	0.35
Wage	0.53	0.35	0.67	0.48	0.72	0.91	0.65	0.48

[1] All data are in logs and filtered using the H-P filter with the smoothing parameter of 1600.
[2] Relative to the standard deviation of output.
[3] Based on author's simulation of Andolfatto's (1996) model. Original calibration is used.

of the standard RBC model with GHH preference, and those of the complete-markets RBC model with labor market frictions, developed by Andolfatto (1996). For the standard RBC model, the parameters are calibrated to the same targets as in the baseline model wherever feasible. For the model by Andolfatto (1996), his calibration is used. However, notice that the stochastic process for the TFP shock, the only source of business cycle fluctuations in all models, is the same for all models, including the baseline model.

Below I will evaluate the cyclical properties of the baseline model in the same order in which the U.S. business cycle properties are described in Section 3.

1. **(Output)** The baseline model replicates the size of the volatility and the persistence of output in the data. The standard deviation of output of the baseline model (1.51 percent) is very close to that of the U.S. economy (1.58 percent). Since the shock to TFP that is fed into the model is estimated from the data, the closeness implies that the model has an amplification mechanism that is as strong as the one in the U.S. economy. Moreover, autocorrelation of output is 0.73 in the baseline model, which is lower than 0.84 in the U.S.

economy but still strong. Compared with the baseline model, the standard real business cycle model with the same GHH preference generates a lower volatility of output (1.19 percent) but a similar autocorrelation (0.72). The difference from the baseline model is due to the lack of the extensive margin of labor supply adjustments. Output volatility is close to that in the model by Andolfatto (1996). However, the model by Andolfatto (1996) exhibits an autocorrelation (0.81) that is closer to the data. It is because the labor market adjustment lags the cycle by the timing assumption, as discussed. If the timing of the baseline model is changed to be the same as in Andolfatto (1996), the baseline model can also show a similar autocorrelation.

2. **(Consumption)** Consumption in the baseline model economy replicates the key properties of the U.S. business cycle data; consumption is less volatile than output and strongly positively correlated with output. However, quantitatively, the volatility of consumption relative to the output volatility in the model (0.50) is smaller than the level of the U.S. economy (0.80). In other words, the model exhibits an excess smoothness of consumption. However, this is due to comparing the consumption in the model with total consumption in data, which includes durable consumption expenditures. The relative volatility of consumption of nondurable goods and services is also about half in data. As shown in Table 7, consumption volatility relative to output volatility is exactly the same, at 0.50, in the standard RBC model, and even lower (0.28) in Andolfatto's (1996) model. The persistence of consumption in the baseline model is weaker than in the U.S. economy. The autocorrelation is 0.74 in the baseline model and 0.83 in the U.S. economy. This basically echoes the lower persistence of output in the baseline model.

3. **(Investment)** Investment in the baseline model replicates the key cyclical properties of the U.S. data; investment is substantially more volatile than output and strongly procyclical. However, the volatility in the baseline model is slightly weaker than in the data; the standard deviation of investment relative to that of output is 4.64 in the data and 3.45 in the model. The standard RBC model and Andolfatto's (1996) model exhibit similar success in replicating these cyclical properties.

4. **(Unemployment and vacancies)** As extensively discussed in the previous sections, the baseline model generates the observed high volatility of unemployment and vacancies. The models by Andolfatto (1996) and Merz (1995) failed in this dimension. The baseline model also generates strong countercyclicality of unemployment (correlation of -0.97 in the model and -0.84 in the data) and strong procyclicality of vacancies (correlation of 0.93 in the model and 0.89 in the data).

5. **(Market tightness)** The baseline model replicates the observed high volatility and the strong procyclicality of the market tightness as well. Moreover, the baseline model, unlike those by Andolfatto (1996) and Merz (1995), successfully replicates the strong negative correlation between unemployment and vacancies.

6. **(Job finding rate)** The baseline model replicates the observed large volatility and strong procyclicality of the job finding rate. Correlation with output is high both in the baseline model (0.97) and in the U.S. economy (0.82).

7. **(Aggregate hours)** The baseline model generates a lower volatility of aggregate hours than in the data, but replicates its strong procyclicality. The volatility of the aggregate hours is about the same as the output volatility in the U.S. data, while it is about two-thirds in the baseline model. The contemporaneous correlation between output and aggregate hours is 0.99 in the baseline model and 0.87 in the U.S. economy. However, while the aggregate hours lag the cycle by one quarter in the U.S. economy, there is no lag for the baseline model. Correlation with a quarter lag (correlation between output and aggregate hours one quarter later) remains high at 0.89 in the U.S. data but drops to 0.77 in the baseline model. As I discussed, the model lacks strong persistence in vacancies posting and other macroeconomic aggregates. Lack of lag of the aggregate hours and weaker volatility of aggregate hours are shared by Andolfatto's (1996) model and the standard RBC model.

8. **(Employment and average hours)** While the standard RBC model includes only one margin of labor supply adjustments, the fluctuations of aggregate hours can be broken down into the extensive and the intensive margins in the baseline model economy as well as in Andolfatto's (1996) model. The standard deviations of employment (extensive margin) and average hours worked (intensive margin) are 0.43 and 0.20, respectively, in the baseline model. In the U.S. economy the volatilities are 1.05 and 0.51. They are higher in the U.S. economy, but the relative size is similar in both economies; the volatility of the extensive (intensive) margin is two-thirds (one-third) of the volatility of the aggregate hours. The model by Andolfatto (1996) generates a similar relative size of volatility.

   An important difference between the extensive and intensive margins in the data is that employment lags the cycle by a quarter, while the average hours do not lead or lag the cycle. In the U.S. economy, the contemporaneous correlation between output and employment is 0.80, whereas the correlation between output and employment in the next quarter is higher at 0.85. Contemporaneous correlation between output and average hours is 0.72 in the U.S. economy. The baseline model fails to generate the lag of employment; both employment and average hours do not lead or lag the cycle, although employment shows stronger persistence. Correlations between output and contemporaneous and lagged employment are 0.94 and 0.80, respectively, in the baseline. The correlations for the average hours are 0.97 and 0.63 in the model. The model by Andolfatto (1996) successfully generates the lag of employment by a quarter and no lead or lag for average hours. However, as I discussed, it is due to the timing assumption which delays the employment adjustment by a quarter. If the same timing assumption is used in the baseline model, the model can replicate the lead and lag structure as well.

9. **(Labor share)** Another advantage of the current model over the standard RBC model is that labor share fluctuates over the business cycles in the current model, while labor share of income is constant (equal to labor share parameter of the production function with the standard Cobb-Douglas production function) over the business cycles in the standard RBC model. Changes in the labor share are also important in generating the different cyclical properties between productivity and the real wage, because the average wage is productivity times labor share in the model. The baseline model replicates the features that labor share is less volatile than output and is countercyclical. However, quantitatively,

the properties of the model economy do not match well their counterparts in the data; in the model economy, the standard deviation of labor share is 0.41 percent, which is lower than in the data (0.77 percent). The contemporaneous correlation between output and labor share is -0.97 in the baseline model, which is substantially stronger than in the U.S. economy (-0.19). The cyclical properties of labor share in the model by Andolfatto (1996) are somewhere between the baseline model and the standard RBC model (constant labor share). In particular, the volatility of labor share is even weaker (relative standard deviation is 0.11 compared with 0.27 in the baseline model) in his model.[14]

10. **(Productivity and wages)** Models with labor market frictions can generate a diversion between productivity and wages, which the standard RBC model cannot. The baseline model replicates that both productivity (measured by output per hour) and wages (measured by compensation per hour) are less volatile than output, and both are procyclical. Quantitatively, however, the volatilities are similar between the two in the data, while productivity is more volatile in the model; the volatilities of productivity and wages relative to output volatility are 0.50 and 0.57 in the data, while they are 0.41 and 0.14 in the baseline model. Volatility of wages is smaller in the model because the labor share is moving in exactly the opposite direction of productivity over the business cycles, while the negative correlation is not strong in the data. Regarding the cyclicality, both productivity and wages are too strongly procyclical in the model. Output per worker and compensation per worker are both strongly procyclical and less volatile than output, in both the data and the model. The relative volatilities of productivity (0.42) and wage (0.35) in Andolfatto's (1996) are closer to data, thanks to a small volatility of the labor share. Both productivity and wages are strongly procyclical in his model as well.

8 Conclusion

I combine the incomplete-markets real business cycle model with the Mortensen-Pissarides labor market search and matching model and investigate the cyclical properties of the model. I find that the baseline model can replicate the observed high volatility of unemployment and vacancies and their strong negative correlation. The success of the current model over the previous models, including those with labor market frictions, is parallel to the success of the Mortensen-Pissarides model calibrated by Hagedorn and Manovskii (2008a) compared with the same model calibrated by Shimer (2005). The labor-leisure choice plays a crucial role for this main result in two ways. First, because of the additional utility from leisure, the baseline model has a strong amplification effect without relying on a very high replacement rate of unemployment insurance benefits, as in Hagedorn and Manovskii (2008a). The replacement rate of 0.64 in the baseline model can generate as strong an amplification effect as in Hagedorn and Manovskii (2008a) with a much higher replacement rate of 0.955. In other words, the baseline calibration implies that utility from leisure is as valuable as approximately 30 percent of the replacement rate. Second, the labor-leisure choice adds an extra amplification through the intensive margin of labor supply adjustments. I also investigate the role of market incompleteness, by comparing the cyclical

[14] Choi and Ríos-Rull (2009) further investigate cyclical properties of labor share generated by the real business cycle models with labor market frictions.

properties of the baseline model with the model with a relaxed borrowing limit, and the model with varying degree of individual productivity shocks. I find that these features do not affect the cyclicality of unemployment and vacancies in a sizable manner for the same reason that the cyclical properties of the incomplete-markets model by Krusell and Smith (1998) are similar to those in its complete-markets counterpart: Workers in the model, on average, accumulate enough assets so that self-insurance is sufficient for the model to behave like its complete-markets counterpart in terms of aggregate variables. The baseline model is also found to improve upon existing models in replicating business cycle properties, mainly through a stronger amplification and having both margins of labor supply adjustments.

A variety of future extensions are of interest. First, the performance of the model, especially the persistence of macroeconomic variables, could be improved by modeling the number of vacancies as stock instead of flow, as in Fujita and Ramey (2007). Second, since the model developed in this paper is built on the standard incomplete-markets general equilibrium model, the model can be easily used to investigate business cycle and welfare implications of different labor market policies. Third, the model can be used to revisit the question of the cost of eliminating business cycles and the related analysis of countercyclical fiscal policy. Finally, by incorporating nominal frictions, the extended model can be used to analyze effects of monetary policy on heterogeneous workers.

Appendix A Details of Computation

A.1 Approximate Equilibrium

Krusell and Smith (1998) find that the approximate equilibrium where the aggregate capital stock instead of the distribution of capital holding is used as an aggregate state variable provides a good approximation of the true equilibrium where the distribution itself is an aggregate state variable. Following their insight, the type distribution of workers, x, is represented by the aggregate capital stock held by workers K, and the total measure of employed workers N. Representing x, which is an infinitely dimensional object, by a set of its statistics (K, N) makes it feasible to store the functions of x in a computer. I will show that using (K, N) is sufficient to obtain a good approximation of the true equilibrium. Notice that, as usual, there is a trade-off between accuracy and computational cost (time). It is usually better to use more statistics associated with x to improve the accuracy of the approximation. Intuitively, if agents are allowed to use more information about x, the decision of agents would be closer to the decision in the true equilibrium where agents can use x when making decision. On the other hand, it takes more time to solve the model when more statistics are used to represent x. My choice of (K, N) to represent x is based on the fact that it takes a long time to solve the model even if a small set of statistics is used.

Using the aggregate state variables (z, K, N), the workers' value function, and the optimal decision rules are $W(z, K, N, e, s, a)$, $g_a(z, K, N, e, s, a)$, and $g_c(z, K, N, e, s, a)$, respectively. The firms' value functions are $J(z, K, N, s, a)$ and $\widetilde{J}(z, K, \widetilde{N})$. Notice that $\widetilde{N}$ corresponds to $\widetilde{x}$. The bargaining outcome is a pair of functions $\ell(z, K, N, s, a)$ and $w(z, K, N, s, a)$.

The law of motion for the type distribution of workers, $\widetilde{f}_x(z, x)$ and $f_x(z, \widetilde{x})$, are replaced by a pair of laws of motion, $\varphi_K(z, K, N)$ and $\varphi_{V/S}(z, K, \widetilde{N})$, where V/S is market tightness. Given the current $\widetilde{N}$, market tightness V/S is sufficient to compute the size of employment after the matches are created and destroyed, N. In other words, I use the forecasting function for market tightness instead of the forecasting function for N. For completeness, the law of motion for (K, N) can be expressed as follows:

$$(39) \quad K' = \varphi_K(z, K, N)$$

$$(40) \quad N = (1 - \lambda)\widetilde{N} + f_m(S, S\varphi_{V/S}(z, K, \widetilde{N}))$$

where $S = 1 - \widetilde{N} + \lambda\widetilde{N}$. It is easy to see that $\widetilde{N}' = N$.

To properly define the workers' problem, it is necessary to give three more forecasting functions: $\varphi_{K/Y}(z, K, N)$, $\varphi_d(z, K, N)$, and $\varphi_t(z, K, N)$. These are forecasting functions for the capital-output ratio, dividends, and lump-sum transfer from the unemployment insurance program, respectively. Using the forecasting function for the capital-output ratio, agents can form expectations for the interest rate $r(z, K, N)$ and labor productivity $p(z, K, N)$. The forecasting function for dividends and the government lump-sum transfer directly give $d(z, K, N)$ and $t(z, K, N)$, respectively. The definition of the recursive equilibrium with partial information can be constructed by basically replacing x with (K, N) in the definition of the original equilibrium.

A.2 Data Structure

The value function of workers $W(z, K, N, e, s, a)$ is interpolated in the dimension of continuous states (K, N, a). Piecewise-linear interpolation is used for each of K and N, while a spline interpolation is used for a. The optimal decision rules of workers, $g_a(z, K, N, e, s, a)$ and $g_c(z, K, N, e, s, a)$, are interpolated as well. Piecewise-linear interpolation is used for each of (K, N) and a, respectively. As for the value of the firms, $J(z, K, N, s, a)$ and $\widetilde{J}(z, K, \widetilde{N})$, piecewise-linear interpolation is used for each of the aggregate states and the individual asset holding. The bargaining outcomes $w(z, K, N, s, a)$ and $\ell(z, K, N, s, a)$ are interpolated using piecewise-linear interpolation for the aggregate states (K, N) as well as the asset holding a. The forecasting functions for aggregate capital in the next period, $\varphi_K(z, K, N)$, market tightness, $\varphi_{V/S}(z, K, \widetilde{N})$, capital-output ratio, $\varphi_{K/Y}(z, K, N)$, dividends, $\varphi_j(z, K, N)$, and government transfer, $\varphi_t(z, K, N)$, are parameterized using the following log-linear functional forms. Notice that d and t are not in log because they can take a negative value. For ease of notation, the set of all the coefficients associated with the five forecasting functions defined below is denoted as Φ.

$$\log K' = \log \varphi_K(z, K, N) = \Phi_z^{K,0} + \sum_{i=1}^{I_K} \Phi_z^{K,K,i} (\log K)^i + \sum_{i=1}^{I_N} \Phi_z^{K,N,i} (\log N)^i \tag{41}$$

$$\log \frac{V}{S} = \log \varphi_{V/S}(z, K, \widetilde{N}) = \Phi_z^{V,0} + \sum_{i=1}^{I_K} \Phi_z^{V,K,i} (\log K)^i + \sum_{i=1}^{I_N} \Phi_z^{V,N,i} (\log \widetilde{N})^i \tag{42}$$

$$\log \frac{K}{Y} = \log \varphi_{K/Y}(z, K, N) = \Phi_z^{Y,0} + \sum_{i=1}^{I_K} \Phi_z^{Y,K,i} (\log K)^i + \sum_{i=1}^{I_N} \Phi_z^{Y,N,i} (\log N)^i \tag{43}$$

$$d = \varphi_d(z, K, N) = \Phi_z^{d,0} + \sum_{i=1}^{I_K} \Phi_z^{d,K,i} (\log K)^i + \sum_{i=1}^{I_N} \Phi_z^{d,N,i} (\log N)^i \tag{44}$$

$$t = \varphi_t(z, K, N) = \Phi_z^{t,0} + \sum_{i=1}^{I_K} \Phi_z^{t,K,i} (\log K)^i + \sum_{i=1}^{I_N} \Phi_z^{t,N,i} (\log N)^i \tag{45}$$

The functional forms above are slightly more general than those used in Krusell and Smith (1998), because the forecasting function above can include higher order terms associated with $\log K$ and $\log N$. Put differently, the functional forms used in Krusell and Smith (1998) are the special case where $I_K = I_N = 1$. I find that there is a sizable improvement in terms of accuracy measured by adjusted R^2 by increasing $I_K = I_N$ from 1 to 2. Since increasing $I_K = I_N$ above 2 makes the convergence hard to obtain, I choose $I_K = I_N = 2$ for all model economies. I also found that including cross-terms of $\log K$ and $\log N$ does not improve the fit of the forecasting functions substantially. Finally, the type distribution of workers is approximated by storing probability measure over $\widetilde{X} \equiv E \times S \times \widetilde{A}$ where $\widetilde{A}$ is the discretized state space of the original asset space A.

A.3 Algorithm

The following algorithm, which is based on Krusell and Smith (1998), is used to solve the model. For the initial condition for the value function of workers and firms, those obtained from the steady-state economy are used.

1. Set a guess for the coefficients of the forecasting functions Φ_0.
2. Set a guess for the bargaining solution $w_0(z, K, N, s, a)$. Notice that $\ell(z, K, N, s, a)$ can be easily computed from the associated first-order condition, with the GHH utility function.
3. Given Φ_0, $w_0(z, K, N, s, a)$, and $\ell(z, K, N, s, a)$, solve the workers' optimization problem. This step includes the following sub-steps:
   (a) Set a guess for workers' value function $W_0(z, K, N, e, s, a)$.
   (b) Given the future value $W_0(z, K, N, e, s, a)$, bargaining solution, and forecasting functions with coefficients Φ_0, update the value function using the workers' Bellman equation. Denote the updated value function $W_1(z, K, N, e, s, a)$.
   (c) Compute a distance between $W_0(z, K, N, e, s, a)$ and $W_1(z, K, N, e, s, a)$ using some norm (sup-norm is used).
   (d) If the distance is smaller than the predetermined tolerance level, this stage is done. Otherwise, update the value function using $W_0(z, K, N, e, s, a) = W_1(z, K, N, e, s, a)$ and go back to step (b).
   (e) To speed up the iteration procedure, I use Howard's algorithm.
4. Given Φ_0 and $w_0(z, K, N, s, a)$, solve the firms' optimization problem. This step includes the following sub-steps. Notice that $\widetilde{J}(z, K, \widetilde{N}) = 0$ and thus there is no need to solve it.
   (a) Set a guess for the firms' value function $J_0(z, K, N, s, a)$.
   (b) Given the future value $J_0(z, K, N, s, a)$, bargaining solution, and forecasting functions with coefficients Φ_0, update the value function using the Bellman equation for the firm. Denote the updated value function $J_1(z, K, N, s, a)$.
   (c) Compute a distance between $J_0(z, K, N, s, a)$ and $J_1(z, K, N, s, a)$ using some norm (sup-norm is used).
   (d) If the distance is smaller than the predetermined tolerance level, this stage is done. Otherwise, update the value function using $J_0(z, K, N, s, a) = J_1(z, K, N, s, a)$ and go back to step (b).
5. Using the value functions and the optimal decision rules for workers and the firms obtained in the last two steps, solve the bargaining problem, and update the bargaining solution. Again, $\ell(z, K, N, s, a)$ can be computed easily and there is no need for iteration. Denote the obtained surplus sharing rule as $w_1(z, K, N, s, a)$.
6. Compute a distance between $w_0(z, K, N, s, a)$ and $w_1(z, K, N, s, a)$ using some norm (sup-norm is used).

7. If the distance is smaller than the predetermined tolerance level, this stage is done. Otherwise, update the surplus sharing rule and go back to step 3. The surplus sharing rule is updated as follows, with the parameter ξ_w controlling the speed of updating.

$$w_0(z,K,N,s,a) = (1-\xi_w)w_0(z,K,N,s,a) + \xi_w w_1(z,K,N,s,a) \quad (46)$$

8. Using the optimal decision rules and the bargaining solution obtained from the previous steps, run a simulation of T periods (I use $T = 10500$). This step involves the following sub-steps:

   (a) Draw a sequence $\{z_t\}_{t=1}^{T}$, using the Markov transition probability $\{\pi^z_{z,z'}\}$.
   (b) Set $t = 1$. Set an initial distribution of workers $x_t(e,s,a)$. The stationary distribution of the steady-state version of the model is used as the initial $x_t(e,s,a)$.
   (c) Using the current type distribution $x_t(e,s,a)$, compute aggregate statistics $\{K_t, N_t\}$.
   (d) Using the value functions and the optimal decision rules of the workers and the firms, compute $\left\{\frac{V_t}{S_t}, \frac{K_t}{Y_t}, d_t, t_t\right\}$.
   (e) Using the optimal decision rules of workers and the number of posted vacancies consistent with the forecasting functions with the coefficients Φ_0, update the type distribution and obtain $x_{t+1}(e,s,a)$.
   (f) If $t = T$, stop. Otherwise, set $t = t+1$ and go back to step (c).

9. The simulation generates an artificial time series of $\left\{K_t, N_t, \frac{V_t}{S_t}, \frac{K_t}{Y_t}, d_t, t_t\right\}_{t=1}^{T}$. Drop the observation of the first $T_0 = 500$ periods and keep the remaining $T_1 = 10000$ periods. Use OLS regressions to obtain the set of coefficients Φ_1 that best fit the simulated data series.
10. Compute a distance between Φ_0 and Φ_1 using some norm (sup-norm is used).
11. If the distance is smaller than the predetermined tolerance level, the iteration is done. Otherwise, update Φ and go back to step 2. Use the following formula to update Φ. In updating Φ, a conservative updating (low value of ξ_ϕ) turns out to be necessary for the stable implementation of the algorithm, at the cost of longer computational time.

$$\Phi_0 = (1-\xi_\phi)\Phi_0 + \xi_\phi \Phi_1 \quad (47)$$

A.4 Accuracy

One way to measure the accuracy of the approximated equilibrium is an adjusted R^2, which is summarized in Table 8. The R^2 for the dividends d is relatively low because x instead of $\widetilde{x}$ is used for the forecasting function, which helps to reduce the number of aggregate states. However, since the size of dividends is very small, the forecasting error of the dividend function does not significantly affect the main results.

Table 8: Accuracy of the approximate equilibrium: baseline model economy

	Adjusted R^2				
Regressand	z_1	z_2	z_3	z_4	z_5
K'	0.9999999	0.9999997	0.9999991	0.9999996	0.9999995
K/Y	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
V/S	0.9990510	0.9991965	0.9997627	0.9998746	0.9999166
d	0.9864461	0.9876701	0.9834568	0.9689151	0.9743233
t	0.9999999	0.9999999	0.9999998	0.9999998	0.9999997

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