Rising Indebtedness and Temptation: A Welfare Analysis[*]

Makoto Nakajima[†]

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Abstract

Is the observed large increase in consumer indebtedness since 1970 beneficial for U.S. consumers? This paper quantitatively investigates the macroeconomic and welfare implications of relaxing borrowing constraints using a model with preferences featuring temptation and self-control. The model can capture two contrasting views: the positive view, which links increased indebtedness to financial innovation and thus better consumption smoothing, and the negative view, which is associated with consumers' *over-borrowing*. I find that the latter is sizable: the calibrated model implies a social welfare *loss* equivalent to a 0.4 percent decrease in per-period consumption from the relaxed borrowing constraint consistent with the observed increase in indebtedness. The welfare implication is strikingly different from the standard model without temptation, which implies a welfare *gain* of 0.7 percent, even though the two models are observationally similar. Naturally, the optimal level of the borrowing limit is significantly tighter according to the temptation model, as a tighter borrowing limit helps consumers by preventing over-borrowing.

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[†]Research Department, Federal Reserve Bank of Philadelphia. Ten Independence Mall, Philadelphia, PA 19106. E-mail: makoto.nakajima@phil.frb.org.
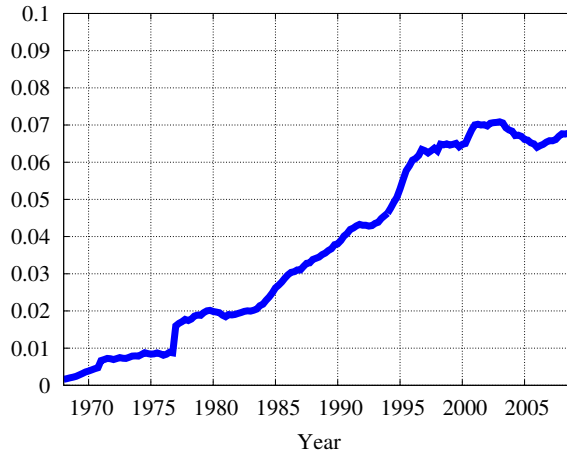
Figure 1: Total unsecured consumer debt over GDP

1 Introduction

Since the 1970s, there has been a substantial increase in the indebtedness of U.S. consumers, although that trend might reverse as a result of the recent downturn. Total household debt in the U.S. increased from 43 percent of GDP in 1982 to 62 percent in 2000. Both unsecured and secured debt increased. Figure 1 shows the trend of unsecured consumer debt relative to GDP.[1] It was close to zero before 1970 but has gradually increased since then, and it has stabilized around 7 percent since 2000. While an increase in indebtedness is often seen as a result of an innovation in the financial sector and thus is linked to a gain in social welfare, there are two channels through which rising indebtedness is associated with a welfare loss. First is the general equilibrium effect; increased indebtedness might induce under-saving, which slows down capital accumulation. Second, there is a popular perception that consumers might be *over-borrowing* and *over-consuming*. While the first channel is studied, among others, by Campbell and Hercowitz (2009) and Obiols-Homs (2011), the second channel has not been studied, since it cannot be systematically captured by models with the standard exponential preferences. This paper intends to fill the void.

In order to analyze over-borrowing and over-consuming, I introduce preferences featuring temptation and self-control, which is developed by Gul and Pesendorfer (2001, 2004a,b). Specifically, I use a version of the macroeconomic model with the temptation preferences, which is developed by Krusell et al. (2010). In the model, consumers are *tempted* to borrow and consume more than they would choose if they could exert perfect *self-control*. Therefore, this framework is naturally

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[1] Total household debt is computed by Smith (2009). Unsecured consumer debt is measured as the revolving consumer credit in the G.19 series of the Federal Reserve Board (FRB). In the FRB data, total consumer credit consists of non-revolving and revolving credit. Revolving credit mainly consists of loans for automobiles, mobile homes, and boats but also includes some unsecured credit. Livshits et al. (2010) constructed an unsecured consumer credit data series that includes not only revolving credit but also a part of non-revolving credit. However, the difference between the revolving credit and the unsecured consumer credit they constructed is small (less than one percentage point as a percentage of disposable income) for the period for which more reliable data are available (after 1989).

suitable in studying over-borrowing and over-consuming in response to a relaxed borrowing constraint induced by an innovation in the financial sector. There are supporting evidences – based on both survey results and estimated structural models – that consumers face a temptation and self-control problem, which supports the use of the temptation model for the analysis.

There are three main findings. First, not only are the models with and without temptation *observationally similar* in the steady-state equilibrium, as shown in Angeletos et al. (2001), the aggregate response associated with an increased indebtedness, which is induced by a relaxed borrowing limit, is both qualitatively and quantitatively similar between the two models. Angeletos et al. (2001) compare the macroeconomic implications of models with and without temptation and argue that the temptation model replicates various dimensions of consumption and savings behavior better than the standard model without temptation, although both models are observationally similar in terms of the average life-cycle profile of aggregate saving.[2] Furthermore, Barro (1999) shows the observational equivalence between the neoclassical growth models with and without temptation. My findings echo and extend theirs: models with and without temptation have similar macroeconomic implications. But how about welfare implications? This is the key issue investigated in this paper.

Indeed, I find that, in spite of the observational similarity, the models with and without temptation have strikingly different welfare implications. This is the second main finding. According to the calibrated model, while a relaxed borrowing limit is associated with a social welfare gain equivalent to a 0.7 percent increase in flow consumption in the model without temptation, the temptation model implies a welfare *loss* of 0.4 percent. The difference is due to the over-borrowing by consumers in response to a relaxed borrowing limit. The problem is serious from a policy perspective because the models with and without temptation are hard to distinguish but have contrasting welfare implications. Barro (1999) argues that we can largely keep relying on the neoclassical growth model with exponential discounting consumers as the workhorse framework, even though there is some evidence in favor of temptation, because the growth models with the two different preference specifications are observationally equivalent. The case I study in this paper shows that one needs to be careful even if the temptation model is observationally similar to the standard model without temptation, because the two models could have very different implications on welfare, and the optimal policy.

Finally, I find that the optimal level of the borrowing limit is tighter, at about 7 percent of average income, in the model with temptation compared with the standard model without temptation, whose optimal borrowing limit is about 19 percent. Even in a standard no-temptation model like those in Campbell and Hercowitz (2009) and Obiols-Homs (2011), there is a threshold level of the borrowing limit above which the gain from a relaxed borrowing limit (better consumption smoothing) is dominated by the negative general equilibrium effect (capital decumulation). In the model without temptation, consumers suffer from a relaxed borrowing limit if the limit is already above 19 percent of average income. In the model with temptation, the limit is substantially

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[2] While both Angeletos et al. (2001) and Barro (1999) conduct their analysis based on the hyperbolic discounting model, the hyperbolic discounting model is a special case of the temptation model used in this paper. Therefore, their findings are applicable for the temptation model used in this paper. See Section 2.9 for more detailed discussion.

lower, at 7 percent. The reason why the optimal borrowing limit is substantially lower in the temptation model is over-borrowing. When consumers are subject to temptation, there is a potential for extra welfare gain from restricting borrowing of consumers.

This paper sits at the intersection of two strands of literature. The first is associated with the model of consumers tempted to over-consume.[3] The idea of over-consumption is first formalized by Strotz (1956). Phelps and Pollak (1968) use the quasi-hyperbolic discounting function in the context of intergenerational time preferences. Laibson (1997) embeds the quasi-hyperbolic discounting preferences into the standard life-cycle model and studies the role of illiquid assets like housing as a commitment device. Laibson et al. (2007) use simulated method of moments to jointly estimate key parameters associated with the quasi-hyperbolic discounting model. Krusell et al. (2010) extend the model with the Gul-Pesendorfer preferences featuring temptation and self-control to the macroeconomic general-equilibrium model. I use the temptation model of Krusell et al. (2010) instead of the hyperbolic discounting model for two reasons. First, the temptation model is more general and includes the quasi-hyperbolic discounting model as a special case. Second, the temptation model enables more straightforward welfare analysis than the hyperbolic discounting model, in which a consumer is modeled as consisting of "multiple selves" with different utility functions. Krusell et al. (2010) use the temptation model and show that a savings subsidy (or negative capital income tax) is optimal in the neoclassical growth model with temptation, while it is optimal not to tax capital income in the no-temptation model.

The second strand of literature is associated with macroeconomic models with incomplete markets. The model developed in this paper is built on a general equilibrium model with incomplete markets initially developed by Huggett (1996) and Aiyagari (1994). The current paper is especially related to Campbell and Hercowitz (2009) and Obiols-Homs (2011); both use a general equilibrium model with incomplete markets to investigate a cross-section of the welfare consequences associated with rising debt in the U.S., but both use the standard exponential discounting preferences. This paper introduces preferences featuring temptation and self-control into the life-cycle general equilibrium model with incomplete markets. In this sense, the model developed in this paper is closest to the one in İmrohoroğlu et al. (2003); they study macroeconomic and welfare effects of having an unfunded Social Security program in the life-cycle general equilibrium model with hyperbolic discounting consumers. However, their focus is not on indebtedness or market incompleteness. In addition, this paper is the first one to solve for the equilibrium transition dynamics between steady states in a model with temptation and self-control.

Although the model used for analysis is rich in features, there are limitations. First, the model abstracts from aggregate shocks. Second, I assume that all consumers have the same preferences. For example, in the temptation models, all consumers share the same parameter values associated with temptation. Third, I do not allow any commitment device for consumers. As Laibson (1997) showed, consumers with temptation would optimally try to use commitment devices, if available, to restrain themselves from over-consuming in the future. Examples are durable goods (such

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[3] Angeletos et al. (2001) provide an overview of the literature. Laibson et al. (2003) use a quasi-hyperbolic discounting model to explain the credit card puzzle. Tobacman (2009) investigates the wealth distribution of such a model. Malin (2008) studies welfare implications of mandatory savings floors in the hyperbolic-discounting model.

as housing) or retirement saving instruments (such as individual retirement accounts (IRAs)). Finally, I consider only unsecured debt. I leave these issues for future research.

The rest of the paper is organized as follows. Section 2 presents the model. At the end of the section, I will argue that the model with the Strotz-Laibson hyperbolic discounting preferences is a special case of the temptation model developed by Krusell et al. (2010) and employed in this paper. Section 3 describes how the model is calibrated for quantitative exercises. Section 4 gives an overview of the computational algorithm with which the model is solved. The Supplementary Appendix includes more details of the calibration and the computational algorithm. Section 5 presents the main results of the paper, using a steady-state analysis. Section 6 conducts an analysis explicitly taking into account the equilibrium transition path from an initial steady state to a new one. Section 7 addresses the sensitivity of the main results. Section 8 concludes.

2 Model

The model is based on the general equilibrium life-cycle model of Huggett (1996), with the version of the Gul-Pesendorfer preferences featuring temptation and self-control that is developed by Krusell et al. (2010). After completing the description of the model, in Section 2.9, I will provide an alternative formulation of the consumer's problem based on the Strotz-Laibson hyperbolic discounting preferences, and argue that, when the strength of temptation is taken to infinity, the two formulations generate the identical optimal decision rule. Therefore, all existing macroeconomic implications under the Strotz-Laibson hyperbolic discounting preferences are valid under the temptation preferences.

2.1 Demographics

Time is discrete and starts from 0. In each period, the economy is populated by I overlapping generations of consumers. In period t, a measure $(1+\nu)^t$ of consumers are born. ν is the constant population growth rate. Each generation is populated by a mass of consumers. Consumers are born at age 1 and could live up to age I. An age-i consumer survives to age $i+1$ with probability s_i. With probability $(1 - s_i)$, the consumer dies. I is the maximum possible age, which implies $s_I = 0$. Consumers retire at the fixed age $I_R < I$. Consumers with age $i < I_R$ are called workers, and those with age $i \geq I_R$ are called retirees.

2.2 Preferences

The preferences of consumers are time separable and characterized by a period utility function, two discount factors, δ and β, and another parameter γ. The period utility function $u(c)$ is standard: it is strictly increasing and strictly concave in c. Consumers do not value leisure; there is no labor supply decision. In Section 7, I relax this assumption and introduce a labor-leisure decision as a sensitivity analysis.

δ and β are called the *self-control discount factor* and the *temptation discount factor*, respectively. γ represents the *strength of temptation*. δ is the only discount factor if the consumer can exert perfect self-control and thus is not affected by temptation. In other words, in a special case where the temptation is nonexistent (strength of temptation γ is zero), the model with temptation and self-control preferences reverts to the standard exponential discounting model with δ as the

only discount factor. $\beta < 1$ is the additional discount factor that a consumer is tempted to discount future utility when making a consumption-savings decision. Formal characterization of the consumers' problem is presented in Section 2.7.

2.3 Technology

There is a representative firm that has access to the constant returns to scale production technology in the form of $Y = ZF(K, L)$. Y is output, Z is the level of total factor productivity, K is capital stock, and L is labor supply. Capital depreciates at a constant rate κ per period.

2.4 Endowment

Consumers are born with zero assets. Each consumer is endowed with one unit of time each period. Time is inelastically supplied for work, since leisure is not valued. Labor productivity of a consumer is characterized by $e(i, p)$, where i captures the life-cycle profile of labor productivity, and p is an idiosyncratic shock to labor productivity. p is assumed to have finite support: $p \in \{p_1, p_2, ..., p_N\}$. Each newborn consumer draws its initial p from an i.i.d. distribution where π_p^0 is the probability attached to each p. After the initial p is drawn, p follows a first-order Markov process with $\pi_{p,p'}$ as the transition probability from p to p'.

2.5 Market Arrangements

Capital and labor are traded competitively. Consumers are not allowed to trade state-contingent securities but can save or borrow using asset a ($a < 0$ represents borrowing), subject to a borrowing limit $\underline{a}_t$.

2.6 Government

The government has three roles in the model: (i) running the Social Security program, (ii) collecting a proportional income tax, and (iii) collecting accidental bequests using an estate tax and redistributing the proceeds with a lump-sum transfer.

The government runs a simple pay-as-you-go Social Security program. The government imposes a flat payroll tax with the tax rate of τ_S on all workers and uses the proceeds to finance the Social Security benefits $b_{t,i}$ of current retirees. It is assumed that all retirees receive the same amount ($\bar{b}_t$) of benefits regardless of their age or contribution, and the government budget associated with the Social Security program balances each period. Formally, $b_{t,i} = 0$ for $i < I_R$ and $b_{t,i} = \bar{b}_t$ for $i \geq I_R$.

The government collects a proportional general income tax with the tax rate τ_I. Both capital and labor income are taxed at the same rate. The proceeds are not redistributed or valued by consumers.

Because of the stochastic death, there are accidental bequests in the model. I assume that the government collects all the accidental bequests using an estate tax and redistributes the proceeds equally to the surviving consumers every period. d_t denotes the lump-sum transfer under the program in period t.

2.7 Consumer's Problem

The problem of an age-i consumer with the current productivity shock p and asset position a in period t can be characterized recursively as follows:

$$V_t(i, p, a) = \max_{a' \geq \underline{a}_t} \left[v_t(i, p, a, a') + \gamma \left(\widetilde{v}_t(i, p, a, a') - \max_{\widetilde{a}' \geq \underline{a}_t} \widetilde{v}_t(i, p, a, \widetilde{a}') \right) \right] \tag{1}$$

where

$$v_t(i, p, a, a') = u(c) + \delta \ s_i \sum_{p'} \pi_{p,p'} V_{t+1}(i + 1, p', a') \tag{2}$$

$$\widetilde{v}_t(i, p, a, a') = u(c) + \beta \ \delta \ s_i \sum_{p'} \pi_{p,p'} V_{t+1}(i + 1, p', a') \tag{3}$$

$$c + a' = (a + d_t)(1 + r_t(1 - \tau_I)) + e(i, p)(1 - \tau_I - \tau_S)w_t + b_{t,i} \tag{4}$$

Equation (1) is the Bellman equation. Equations (2) and (3) define the *self-control utility* and the *temptation utility*, respectively. The only difference between the two is, while future utility is discounted by δ in the former, it is discounted by $\beta\delta$ in the latter. Naturally, when $\beta < 1$, the consumer is *tempted* to consume more in the current period when the consumer is maximizing the temptation utility rather than the self-control utility. Equation (4) is the standard budget constraint, with consumption (c) and next-period assets (a') on the left-hand side, and current-period assets (a), transfers (d_t), after-tax interest income ($(a + d_t)r_t(1 - \tau_I)$), after-tax labor income ($e(i, p)(1 - \tau_I - \tau_S)w_t$), and the Social Security benefits ($b_{t,i}$) on the right-hand side. The maximand of the Bellman equation consists of two parts – the self-control utility, and the part that contains the temptation utility. γ determines the relative strength of the latter. $a' = g_t^a(i, p, a)$ is the optimal decision rule associated with the Bellman equation above.

In order to understand this non-standard Bellman equation, let's consider the two extreme cases first. In an extreme case where $\gamma = 0$, the temptation part of the problem drops out completely, and the consumer's problem reverts back to the one with the standard exponential discounting preferences with the discount factor δ. This is interpreted as the case when the consumer has a perfect self-control and thus is not affected by temptation to consume more today rather than in the future. In the other extreme case where $\gamma \to \infty$, the utility-maximizing consumer wants to choose a' that maximizes the temptation utility, as the relative importance of the self-control utility becomes zero. Notice, however, that since the difference between $\widetilde{v}_t(i, p, a, a')$ and $\max_{\widetilde{a}' \geq \underline{a}_t} \widetilde{v}_t(i, p, a, \widetilde{a}')$ becomes zero, the value updated in the Bellman equation is based on the self-control utility, but with a' that maximizes the temptation utility. The intuition is, although the consumer wants to choose a' to maximize the self-control utility, the consumer succumbs to the temptation and chooses a' that maximizes the temptation utility. In an intermediate case where $\gamma \in (0, \infty)$, the consumer chooses a' to balance the two forces; on the one hand, the consumer wants to choose a' that maximizes the self-control utility, which is associated with the discount factor δ. On the other hand, the consumer also wants to consume more today, to maximize the temptation utility with discount factor $\beta\delta$. γ determines the relative strength of the latter.

2.8 Equilibrium

I will first define the recursive competitive equilibrium where the demographic structure is stationary, even though the size of the population is growing at the constant rate ν. Then I will move on to define the steady-state recursive competitive equilibrium, where prices $\{r_t, w_t\}_{t=0}^{\infty}$ and government policy variables $\{\{b_{t,i}\}_{i=1}^{I}, d_t\}_{t=0}^{\infty}$ are constant over time, although the aggregate variables are growing at the population growth rate.

Let $\mathbf{M}$ be the space of an individual state, i.e., $(i, p, a) \in \mathbf{M}$. Let $\mathcal{M}$ be the Borel σ-*algebra* generated by $\mathbf{M}$, and μ the probability measure defined over $\mathcal{M}$. I will use a probability space $(\mathbf{M}, \mathcal{M}, \mu)$ to represent a type distribution of consumers.

Definition 1 (Recursive competitive equilibrium) *Given a sequence of total factor productivity* $\{Z_t\}_{t=0}^{\infty}$*, a sequence of borrowing limits* $\{\underline{a}_t\}_{t=0}^{\infty}$*, and the initial type distribution of consumers* μ_0*, a recursive competitive equilibrium is a sequence of prices* $\{r_t, w_t\}_{t=0}^{\infty}$*, government policy variables* $\{\{b_{t,i}\}_{i=1}^{I}, d_t\}_{t=0}^{\infty}$*, aggregate capital stock and labor supply* $\{K_t, L_t\}_{t=0}^{\infty}$*, value functions* $\{V_t(i, p, a)\}_{t=0}^{\infty}$*, optimal decision rules* $\{g_t^a(i, p, a)\}_{t=0}^{\infty}$*, and the measure after normalization with respect to population growth,* $\{\mu_t\}_{t=0}^{\infty}$*, such that:*

1. *In each period* t*, given the prices and policy variables,* $V_t(i, p, a)$ *is a solution to the consumer's optimization problem defined in Section 2.7, and* $g_t^a(i, p, a)$ *is the associated optimal decision rule.*

2. *The prices* $\{r_t, w_t\}_{t=0}^{\infty}$ *are determined competitively, i.e.,*

$$r_t = Z_t F_K(K_t, L_t) - \kappa \tag{5}$$
$$w_t = Z_t F_L(K_t, L_t) \tag{6}$$

   where

$$K_{t+1} = \frac{1}{1 + \nu} \int_{\mathbf{M}} g_t^a(i, p, a) \ d\mu_t \tag{7}$$
$$L_t = \int_{\mathbf{M}} e(i, p) \ d\mu_t \tag{8}$$

3. *Given the initial measure* μ_0*, the sequence of the measure of consumers* $\{\mu_t\}_{t=0}^{\infty}$ *is consistent with the demographic transition, the stochastic process of shocks, and the optimal decision rules, after normalization with respect to population growth in each period* t*.*

4. *Government satisfies the period-by-period budget constraint with respect to the Social Security program in each period* t*, i.e.,*

$$\int_{\mathbf{M}} b_{t,i} \ d\mu_t = \int_{\mathbf{M}} e(i, p) \ w_t \ \tau_S \ d\mu_t \tag{9}$$

5. *Government satisfies the period-by-period budget constraint with respect to the estate taxes and the lump-sum transfers in each period t, i.e.,*

$$\int_{\mathbf{M}} d_{t+1} \, d\mu_{t+1} = \frac{1}{1+\nu} \int_{\mathbf{M}} (1 - s_i) \, g_t^a(i, p, a) \, d\mu_t \tag{10}$$

Definition 2 (Steady-state recursive competitive equilibrium) *A steady-state recursive competitive equilibrium is a recursive competitive equilibrium where total factor productivity, the borrowing limit, type distribution, prices, government policy variables, aggregate capital stock and labor supply, the value function, and the optimal decision rule are constant over time, after normalizing the type distribution of consumers by the population growth rate.*

Notice that although I use the word *steady state*, the model is on a balanced growth path with the constant population growth rate and the type distribution of heterogeneous consumers is stationary only after normalization. The measure of consumers is normalized to be a probability measure (total measure is one) each period, which makes all the aggregate variables constant over time instead of growing at the population growth rate.

2.9 Alternative Formulation of the Consumer's Problem with Hyperbolic Discounting

I will provide an alternative formulation of the consumer's problem defined in Section 2.7, based on the Strotz-Laibson hyperbolic discounting preferences. After showing the recursive formulation of the consumer's problem, I will argue that the hyperbolic discounting preferences are the special case of the temptation preferences in terms of allocations; the resulting optimal decision rules are the same as in the problem based on the preferences featuring temptation and self-control with $\gamma \to \infty$.

According to the Strotz-Laibson set-up, the expected lifetime utility of an age-i consumer, U_i, can be defined as follows:

$$U_i = u(c_i) + \beta \, \mathbb{E} \sum_{j=i+1}^{I} \delta^{j-i} u(c_j) \tag{11}$$

In period t, instantaneous utility in period t, $t + 1$, $t + 2$, $t + 3$,..., is discounted by 1, $\beta\delta$, $\beta\delta^2$,... Since β is used only to discount utility from the current period and the next, while δ is used to discount future utility every period, β and δ are called *short-term* and *long-term* discount factor, respectively. Notice that the standard exponential discounting is a special case with $\beta = 1$: in this case, future utility is discounted exponentially at the constant discount factor δ. The important feature of this class of preferences is that the preferences exhibit time inconsistency; the discount factor applied between period $t + 1$ and $t + 2$ in period t is δ, while the discount factor between the same periods changes to $\beta\delta$ in period $t + 1$. In particular, with $\beta \in (0, 1)$, the preferences imply a present bias: if there is no constraint or commitment device, consumers over-borrow and over-consume from the perspective in previous periods.

In the hyperbolic discounting model, the problem of an age-i consumer with the current productivity shock p and asset position a in period t can be characterized by the following Bellman equation:

$$\widetilde{W}_t(i, p, a) = \max_{a' \geq \underline{a}_t} \left[u(c) + \beta \ \delta \ s_i \ \sum_{p'} \pi_{p,p'} W_{t+1}(i+1, p', a') \right] \tag{12}$$

subject to the budget constraint (4). $a' = h_t^a(i, p, a)$ is the optimal decision rule associated with the Bellman equation above. Notice that the value function on the left-hand side, $\widetilde{W}_t(i, p, a)$, is different from the one on the right-hand side, which is $W_t(i, p, a)$. $W_t(i, p, a)$ is obtained by updating the value function with the following equation:

$$W_t(i, p, a) = \left[u(c) + \delta \ s_i \ \sum_{p'} \pi_{p,p'} W_{t+1}(i+1, p', a') \right] \tag{13}$$

where $a' = h_t^a(i, p, a)$ obtained from the Bellman equation (12) and subject to the budget constraint (4).

Intuitively, the consumer chooses the optimal asset level a' with the discounting factor $\beta\delta$ (equation (12)) but the actual value is evaluated with the discount factor δ (equation (13)).[4] This is exactly the problem described in Section 2.7 with $\gamma \to \infty$. In other words, the optimal decision rule $g_t^a(i, p, a)$ obtained from (1) are equivalent to the optimal decision rule $h_t^a(i, p, a)$ obtained from (12). Formally, Proposition 6 of Krusell et al. (2010) proves the equivalence in the neoclassical growth model with a finite horizon.

How about the welfare in the two models? When $\gamma \to \infty$, the value function $V_t(i, p, a)$ obtained in the temptation model (characterized by the Bellman equation (1)) coincides with the value function $W_t(i, p, a)$ obtained in the hyperbolic discounting model (characterized by equation (13)). In other words, the temptation model suggests using just the long-term discount factor δ to discount future utility when evaluating the welfare in the hyperbolic discounting model.[5]

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[4] İmrohoroğlu et al. (2003) distinguish the two cases in terms of what hyperbolic discounting consumers expect about their own future decisions. According to their classification, a *naive* consumer wrongly thinks that future selves make decisions in a time-consistent manner (using only the discount factor δ). On the other hand, a *sophisticated* consumer correctly thinks that future selves are time-inconsistent (using both β and δ). I use the sophisticated consumers, as in Laibson (1996) and Laibson et al. (2007). Angeletos et al. (2001) find that naive and sophisticated hyperbolic discounting consumers behave similarly in their life-cycle model.

[5] However, interpretation of welfare is different in the two models. In the case of the hyperbolic discounting model, as preferences of a consumer change over time, the same consumer at different points of time are interpreted as different "selves." Naturally, the consumer's problem is understood as the dynamic game among multiple selves. On the other hand, in the case of the temptation model, the consumer is modeled as internally compromising between utility from consumption and disutility from exerting self-control against temptation to over-consume each period. Since there is no multiple selves within a consumer in the temptation model, interpretation of welfare is more straightforward.

3 Calibration

This section describes how the steady-state model is calibrated.[6] Consequently, the time script t is dropped throughout the section. Each of the subsections below corresponds to those in Section 2.

3.1 Demographics

One period is set as one year in the model. Age 1 in the model corresponds to the actual age of 20. I is set at 81, meaning that the maximum actual age is 100. I_R is set at 45, implying that consumers retire at the actual age of 65. The population growth rate, ν, is set at 1.2 percent annually. This is the average annual population growth rate of the U.S. over the last 50 years. The survival probabilities $\{s_i\}_{i=1}^{I}$ are taken from the life table in Social Security Administration (2007).[7]

3.2 Preferences

First of all, I assume $\gamma \to \infty$. As I discussed in Section 2.9, the assumption makes the temptation model and the hyperbolic discounting model equivalent in terms of allocation. For the period utility function, the following constant relative risk aversion (CRRA) functional form is used:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{14}$$

σ is set at 1.5, which is a commonly used value. It is also the point estimate of Laibson et al. (2007). Sensitivity of the main results with respect to the value of σ is investigated in Section 7.

Discount factors β and δ are calibrated to be different for different model economies, but the calibration strategy is common. For all economies, I set the temptation discount factor β at first and calibrate the self-control discount factor δ so that the capital-output ratio of the economy in the baseline steady state is 3.0, which is the historical average value of the U.S. economy. In other words, different model economies have different values of discount factors β and δ, but they have the same aggregate capital stock in equilibrium.[8]

In the model without temptation, $\beta = 1$ by assumption. I found that with $\delta = 0.9698$ the steady-state equilibrium of the model generates a capital-output ratio of 3.0. For the model with temptation, I use $\beta = 0.70$ as the baseline value of the temptation discount factor and calibrate δ. The temptation discount factor of 0.70 is the one-year discount factor typically obtained from laboratory experiments.[9] Moreover, the benchmark point estimate of Laibson et al. (2007)

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[6] Additional details of the calibration are found in the Appendix A in the Supplement.

[7] Table 4.C6 of Social Security Administration (2007). An average of the survival probabilities of males and females is used.

[8] Both Angeletos et al. (2001) and Tobacman (2009) calibrate δ for the model without temptation (i.e., exponential discounting model) such that the average wealth holding at age 63 (the age just before retirement) is the same as in the model with temptation (i.e., hyperbolic discounting model) where β and δ are jointly estimated from data. Since the life-cycle profile of asset holdings is similar in the two models, their strategy is close to the strategy adopted in this paper.

[9] Although existing studies estimate parameters with a hyperbolic discounting model in mind, the model is

is $\beta = 0.703$, or the annual discount rate of about 40 percent. The same calibration strategy generates $\delta = 0.9852$. The calibrated value of δ is higher than 0.958, which is the value that Laibson et al. (2007) estimate jointly with β. A large part of the difference is due to the existence of the mortality shock in the current model, which Laibson et al. (2007) do not have. If δ is adjusted by being multiplied by the average survival probability (0.9828), the resulting *effective* δ is 0.968. I also investigate the case when the discount rate is 80 percent annually, which is twice as high as in the baseline temptation model. An 80 percent annual discount rate implies the temptation discount factor of $\delta = 0.56$. Using the same calibration strategy, the economy with $\delta = 0.56$ yields $\delta = 0.9930$.

3.3   Technology

The following standard Cobb-Douglas production function is assumed:

$$Y = ZF(K, L) = ZK^\theta L^{1-\theta} \tag{15}$$

Z is normalized such that, in the baseline steady state, the equilibrium wage is one. The procedure yields $Z = 0.896$. θ is set at 0.36, which corresponds to the average capital share of income of the U.S. economy. The depreciation rate of capital is set at $\kappa = 0.06$ per year. Huggett (1996) calibrates $\kappa = 0.06$ by matching the depreciation-output ratio of the model economy to its empirical counterpart.

3.4   Endowment

I assume the following multiplicative form of individual productivity.

$$e(i, p) = e_i \, p \tag{16}$$

e_i represents the average age-earnings profile and p is the individual productivity shock. Since retirement age is fixed at I_R, $e_i = 0$ for $i \geq I_R$. To calibrate $\{e_i\}_{i=1}^{I_R-1}$, I follow Huggett (1996) and use the data on the median earnings of male workers of different age groups from Social Security Administration (2007).[10] The median earnings data are multiplied by the employment to population ratio of males in each age group. The employment to population ratio for each age group is obtained from McGrattan and Rogerson (2004).[11] Finally, the resulting age-productivity profile is smoothed out by fitting the age profile of the product of the median earnings and the employment to population ratio to a quadratic function of age. The resulting hump-shaped earnings profile can be seen in Figure 2.

The stochastic process for p is calibrated by combining what I call the *bottom 99%*, whose earnings dynamics are captured by the stochastic process of household earnings estimated from the PSID (Panel Study of Income Dynamics) and the *top 1%*, which represent the upper tail of the earnings distribution and are added to replicate the substantial concentration of earnings

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equivalent to the temptation model in terms of allocation and thus the estimated parameters of the hyperbolic discounting model are valid in the temptation model.

[10] The earnings data are taken from Table 4.B6 of Social Security Administration (2007).

[11] Table 3, 4, and 5 of McGrattan and Rogerson (2004).

and wealth in the U.S.[12] It is important that the model captures the observed concentration of earnings and wealth, in order to make sure that the strength of the partial and general equilibrium effects generated by the model is reasonable. As for the stochastic process associated with the *bottom 99%*, I follow the literature and assume that the logarithm of p is initially drawn from a normal distribution $N(0, \sigma_0^2)$ and follows an AR(1) process with the persistence parameter ρ_p and the standard deviation of the innovation term σ_ϵ. The triplet that characterizes the stochastic process is calibrated to $(\rho_p, \sigma_0^2, \sigma_\epsilon^2) = (0.98, 0.30, 0.04)$. The choice is in the middle of estimates in the literature. The persistence parameter ρ_p is estimated to be close to unity in the literature. For example, Storesletten et al. (2004) obtained $\rho_p = 0.9989$, while Huggett (1996) calibrates $\rho_p = 0.96$. The variance of the initial distribution of earnings, σ_0^2, ranges from 0.2735 in Storesletten et al. (2004) to 0.38 in Huggett (1996). σ_ϵ^2 is set so that the life-cycle profile of the earnings variance replicates its empirical counterpart, for example, as shown in Storesletten et al. (2004). The AR(1) process obtained above is approximated using the discretization algorithm of Tauchen (1986).[13]

The *top 1%* is added since the PSID, which is used to estimate the stochastic process of individual productivity shocks often used in the literature, is known to under-sample the top end of the U.S. earnings distribution. The approach employed here corrects such shortcomings by augmenting the estimated stochastic process of earnings with an additional state that captures the *top 1%* of the earnings distribution. In other words, the approach here is a combination of the literature that uses the estimated stochastic process to calibrate the earnings shock and the literature that directly calibrates the earnings shock to capture the high concentration of income and wealth independently from empirically obtained stochastic processes for earnings.[14] Specifically, the *top 1%* is characterized by an additional state of productivity shock, p_1, which is higher than the highest p of the *bottom 99%*. The parameters associated with the *top 1%* are calibrated to satisfy the followings: (i) initially 1% of consumers draw p_1, (ii) the probability that a *bottom 99%* consumer becomes a *top 1%* is set such that the proportion of the *top 1%* among a cohort is always 1%, (iii) the probability of a *top 1%* remaining in the state is 0.92, (iv) when a *top 1%* falls to the *bottom 99%*, the new p is drawn from the ergodic distribution of p among the *bottom 99%*, (v) the level of p_1 is calibrated such that the earnings Gini index of the baseline steady state is 0.61. The probability of remaining a *top 1%* (0.92) is based on Federal Reserve Bank of Dallas (1995), which reports that 47.3 percent of households in the top 1% of income distribution in 1979 remained in the top 1% in 1988.[15] The earnings Gini index of 0.61 is reported in Budría et al. (2002).

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[12] See Budría et al. (2002).

[13] $n_p = 17$ abscissas are used. The abscissas are equally spaced between $-\zeta\sigma_p$ and $\zeta\sigma_p$, where σ_p is the standard deviation of the ergodic distribution of p. Tauchen (1986) chooses $\zeta = 3$ while Huggett (1996) uses $\zeta = 4$. I set $\zeta = 2.1$ so that the life-cycle profile of earnings variances implied by the obtained Markov stochastic process is close to the one implied by the original AR(1) process. In general, for a small n_p, properties of the Markov process obtained using Tauchen's (1986) method vary with the choice of ζ.

[14] A leading example of the latter approach is Castañeda et al. (2003).

[15] $0.92 = 0.473^{\frac{1}{9}}$.

3.5 Market Arrangements

In the baseline steady state, the borrowing limit $\underline{a}$ is set at zero, i.e., there is no borrowing. This assumption corresponds to the fact that there was virtually no unsecured consumer credit in 1970. In experiments, I will relax the borrowing limit to the extent such that the aggregate amount of debt is the same between the model and the corresponding U.S. economy after 1970. In other words, I will back out the degree of relaxation of the borrowing constraint from the observed increase in indebtedness.

3.6 Government

The payroll tax rate for the Social Security contribution τ_S is set at 0.10, which is the average contribution to the Social Security program as a fraction of labor income in the U.S. The proportional income tax rate of $\tau_I = 0.2378$ is set to match the U.S. historical average of the ratio of total (federal, state, and local) government consumption over total income (0.195).

4 Computation

Since there is no analytical solution to the model, the model is solved numerically.[16] The space of asset holdings is discretized, and the choice with respect to asset holdings is also constrained by the discretized state space. The consumer's optimization problem is solved using backward induction. The equilibrium prices (wage and interest rate) and the government policy variables (transfer and Social Security benefits) are found using iteration.

5 Results: Steady-State Analysis

This section presents the main results, based on steady-state comparison. The starting point of the analysis is the economy without debt (i.e., $\underline{a} = 0$), which is calibrated in Section 3. Since this economy mimics the U.S. economy in 1970 at which time unsecured consumer debt was almost nonexistent, I call the economy the *1970 economy*. Next, in order to replicate the increased indebtedness between the 1970s and the 2000s with the model, I assume that the increased indebtedness is due to a relaxed borrowing limit that consumers face. Relaxing the borrowing limit is a parsimonious way to capture various types of innovation in the consumer credit market that happened over the last three decades. The borrowing limit is calibrated such that the aggregate debt is 7 percent of output in the new steady state of the model. Since this level of debt is observed in the U.S. economy in the 2000s, I call it the *2000 economy*. I implement this procedure separately for models with varying degree of temptation. The primary interest is how different macroeconomic and welfare implications are among the models.

Section 5.1 compares the 1970 economy with and without temptation. I show that macroeconomic implications are very similar between the models. This observational similarity result is the reconfirmation of Angeletos et al. (2001). In Section 5.2, macroeconomic implications of increased indebtedness due to the relaxed borrowing limit are investigated, by comparing the 1970 economy and the 2000 economy. I will show that models with and without temptation again exhibit similar responses to the relaxed borrowing limit. Section 5.3 analyzes welfare implications.

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[16] Details of the numerical procedure are found in the Appendix B in the Supplement.

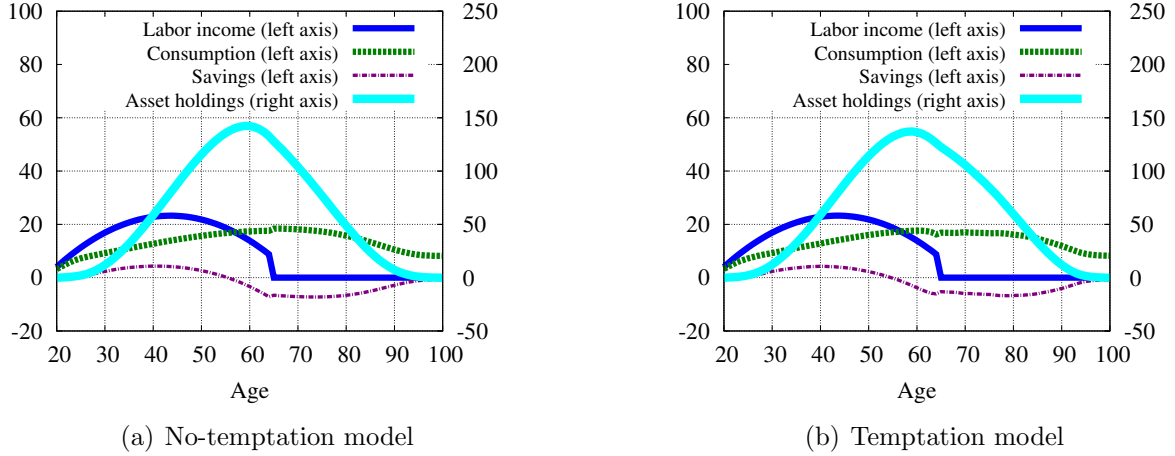(a) No-temptation model   (b) Temptation model

Figure 2: Comparison between models with and without temptation: average life-cycle profiles

The focus is on the difference in the implications between models with and without temptation. If, in addition to the observational similarity, welfare implications of increased indebtedness are also similar between the two models, there is no need to use the non-standard preferences for an analysis of increased indebtedness. What I will show is that this is not the case: although the macroeconomic implications are similar, the welfare implications are substantially different between the models with and without temptation. Finally, in Section 5.4, I investigate the difference in the optimal borrowing limit among models with varying degree of temptation.

5.1 Macroeconomic Implications: The 1970 Economy

Figure 2 compares the average life-cycle profiles of the 1970 model economies without temptation on the left and with temptation (with the temptation discount factor $\beta = 0.70$) on the right. What is most striking is that there is little difference between the two model economies in terms of the average life-cycle profiles. In both economies, the average consumption profile is smoother than the income profile. Consumers save during the working period and dissave during the retirement period. As a result, asset holdings increase until retirement age and decrease after that in both models. Although the temptation model features the temptation discount factor (β), which, ceteris paribus, reduces savings and shifts consumption forward, when the model with temptation is calibrated to generate the same capital-output ratio as in the model without temptation, the self-control discount factor (δ) is calibrated higher in the temptation model. As a result, the effect of the temptation discount factor on the average life-cycle profiles is negated.

The models with temptation and self-control exhibit a slightly higher concentration of wealth, as more consumers are consuming all of their income and saving nothing. The wealth Gini index is 0.786 for the model without temptation, while it is 0.806 for the temptation model with $\beta = 0.70$. The wealth Gini for both economies is not far from 0.803, which is the wealth Gini of the U.S. economy, reported by Budría et al. (2002). For the temptation model with $\beta = 0.56$, the wealth Gini index is 0.820. Tobacman (2009) also compares the wealth inequality implied by the models

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Table 1: Macroeconomic effect of rising indebtedness

Economy[1]	GE[2]	$\underline{a}$[3]	D/Y	K[4]	Y[4]	C[4]	r%	wage	Var(c)[5]
No-temptation model ($\beta = 1.00$)									
1970	–	0.000	0.000	1.000	1.000	1.000	6.00	1.000	0.592
2000	GE	−0.570	−0.070	0.957	0.984	0.991	6.34	0.984	0.580
2000	PE	−0.570	−0.082	0.914	0.968	0.991	6.00	1.000	0.562
Temptation model ($\beta = 0.70$) with $\underline{a}$ of no-temptation model									
1970	–	0.000	0.000	1.000	1.000	1.000	6.00	1.000	0.588
2000	GE	−0.570	−0.085	0.958	0.986	0.990	6.33	0.985	0.577
2000	PE	−0.570	−0.098	0.915	0.969	0.990	6.00	1.000	0.562
Temptation model ($\beta = 0.70$)									
1970	–	0.000	0.000	1.000	1.000	1.000	6.00	1.000	0.588
2000	GE	−0.376	−0.070	0.963	0.986	0.992	6.29	0.986	0.577
2000	PE	−0.376	−0.076	0.928	0.973	0.991	6.00	1.000	0.565
Temptation model ($\beta = 0.56$)									
1970	–	0.000	0.000	1.000	1.000	1.000	5.98	1.000	0.594
2000	GE	−0.297	−0.070	0.964	0.987	0.992	6.26	0.988	0.589
2000	PE	−0.297	−0.074	0.934	0.976	0.991	5.98	1.000	0.580

[1] 1970: Economy with no borrowing. 2000: Economy calibrated to debt-to-output ratio of 7 percent.
[2] GE: general equilibrium. PE: partial equilibrium with prices fixed at the 1970 level.
[3] Borrowing limit relative to total income.
[4] Level in the 1970 (no-debt) economy normalized to one.
[5] Cross-sectional variance of log-consumption, averaged across all age groups.

with and without temptation. In the baseline case with both liquid and illiquid assets, the model with temptation exhibits a Gini coefficient of 0.508, which is slightly higher than the value for the model without temptation (0.488). The magnitude of the difference is comparable to what is obtained here.

5.2 Macroeconomic Implications: Increased Indebtedness

Table 1 summarizes the macroeconomic implications of rising aggregate debt from 1970 to 2000. The first panel (the first three rows) summarizes the results of the standard model without temptation (i.e., exponential discounting model). The first row in each panel shows the levels in the 1970 economy, without debt. The second row is associated with the 2000 steady-state economy. Notice that the general equilibrium (GE) effect is taken into account when the new steady-state equilibrium is obtained. The last row is capturing only the partial equilibrium (PE) effect; the prices (interest rate and wage) are fixed at the 1970 level, but the borrowing limit is relaxed to the 2000 level. By comparing the second (GE) and the third (PE) rows, one can see the strength of the general equilibrium effect in the steady-state economy. In the second panel, the baseline temptation model ($\beta = 0.70$) is employed but the borrowing limit of the no-

temptation model is applied. This panel is intended to highlight the difference in the responses of the two economies when the borrowing limit is relaxed to the same extent. The third panel is associated with the baseline temptation model ($\beta = 0.70$). Notice that the borrowing limit $\underline{a}$ is calibrated to be different from the no-temptation model, but the debt-to-output ratio, which is the calibration target, is the same at 7 percent. In the last panel, results from the temptation model with a lower temptation discount factor ($\beta = 0.56$), when the same calibration strategy as in the first and the third panels is employed, are shown.

As shown in second row of the first panel, the borrowing limit of 57 percent of the average income is needed to generate the aggregate amount of debt as large as 7 percent of output in the model without temptation. In the 2000 economy, the equilibrium capital stock is 4.3 percent lower than in the 1970 economy without borrowing. Since labor is inelastically supplied, the decline in the capital stock generates a decline in output; output and aggregate consumption in the 2000 economy are 1.6 and 0.9 percent lower than in the 1970 economy, respectively. The equilibrium interest rate goes up from 6.00 percent in 1970 to 6.34 in 2000 as capital becomes more scarce, and wage declines by 1.6 percent. A relaxed borrowing limit implies better consumption smoothing. Therefore, the cross-sectional variance of log-consumption averaged across all age groups declines as the borrowing constraint is relaxed: the consumption variance drops from 0.592 in the 1970 economy to 0.580 in the 2000 economy.

What is the role of general equilibrium in shaping the macroeconomic implications discussed above? By comparing the second and third rows, it is clear that, without the general equilibrium effect, macroeconomic responses are stronger. In other words, the general equilibrium effect partly attenuates the macroeconomic responses to the relaxed borrowing limit. Without the general equilibrium effect, both capital stock and output decrease even more, and debt increases more. The consumption variance declines to a larger extent, too.

The second panel in Table 1 summarizes the results for the baseline temptation model ($\beta = 0.70$), but with the borrowing limit obtained for the no-temptation model (0.57 of average income). Most changes are quite similar between the first and second panels. But there is one important difference: the response of aggregate debt is stronger in the temptation model. The borrowing limit of 0.57 of average income increases the debt-to-output ratio to 7 percent in the model without temptation and 8.5 percent in the temptation model.

In the third panel, I implement the same procedure as in the first panel for the baseline temptation model ($\beta = 0.70$). Since the response of aggregate debt to a relaxed borrowing limit is stronger in the temptation model, the borrowing limit that generates a 7 percent debt-to-output ratio is more strict than in the no-temptation model. Indeed, the borrowing limit is calibrated to be 37.6 percent of the average income in the temptation model. The macroeconomic responses to a relaxed borrowing limit in the temptation model are only slightly weaker than in the no-temptation model. Capital stock, output, and consumption decline by 3.7, 1.4, and 0.8 percent, respectively, in the temptation model, while the drops are 4.3, 1.6, and 0.9 percent in the no-temptation model. Cross-sectional consumption variance declines by 1.1 percentage points in the temptation model, compared to a 1.2 percentage point decline in the model without temptation. As in the no-temptation model, the general equilibrium effect partly offsets the macroeconomic responses to the relaxed borrowing limit.

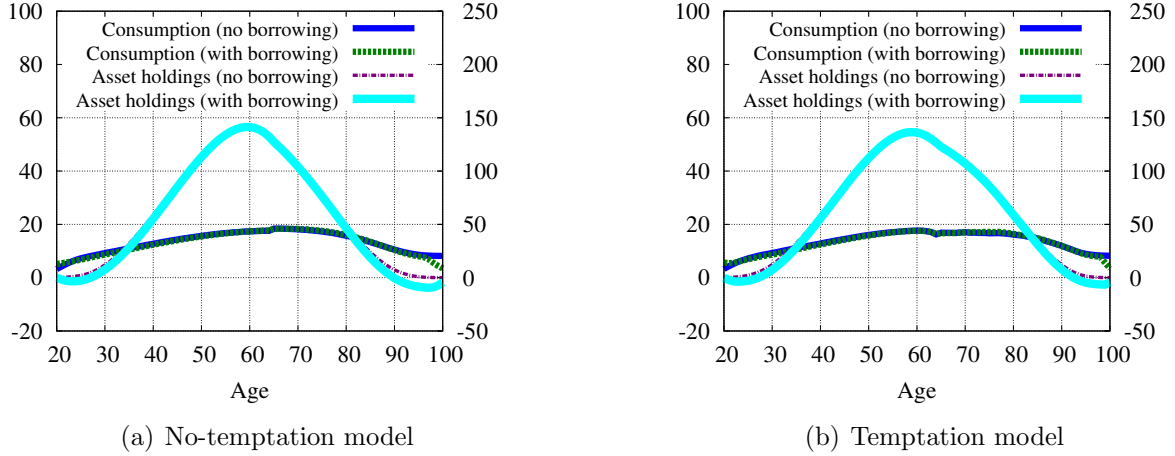(a) No-temptation model   (b) Temptation model

Figure 3: Comparison between models with and without temptation: response of consumption (left axis) and asset holdings (right axis) to a relaxed borrowing limit

Figure 3 compares the 1970 (no borrowing) and 2000 (7 percent debt-to-output ratio) economies with and without temptation. The left panel corresponds to the no-temptation model, and the right panel is associated with the temptation model. Each panel shows how the average life-cycle profiles of consumption and asset holdings react when the borrowing limit is relaxed. The main finding is that the responses of the two models are almost indistinguishable.

The last panel in Table 1 summarizes the macroeconomic implications of an increased indebtedness for the temptation model with a lower temptation discount factor ($\beta = 0.56$). As in the case for the baseline temptation model ($\beta = 0.70$), the borrowing limit is calibrated so that the amount of aggregate debt is 7 percent of output in the new steady state. As the column labeled $\underline{a}$ shows, the borrowing limit has to be even tighter (29.7 percent) than in the baseline temptation model (37.6 percent) because of the stronger response of aggregate debt to a relaxation of the borrowing constraint. The response of macroeconomic aggregates is slightly weaker than in the temptation model with a higher (lower) discount factor (rate). For example, capital stock, output, and consumption decline by 3.6, 1.3, and 0.8 percent, respectively, in the model with $\beta = 0.56$, while the drops are 3.7, 1.4, and 0.8 percent in the baseline temptation model with $\beta = 0.70$.

5.3   Welfare Implications: Increased Indebtedness

In this section, I will investigate the welfare implications of a relaxed borrowing limit. Before starting the analysis, an issue related to the welfare analysis in the current environment needs to be addressed. Since the model used here features a heterogeneous agent model with life-cycle and uninsured idiosyncratic shocks, there is no obvious way to define social welfare. I investigate social welfare in multiple ways. First, I use the ex-ante expected lifetime utility in the steady-state equilibrium as social welfare. The virtue of this welfare criterion is that this naturally takes into account both the welfare gain or loss from changes in aggregate consumption (efficiency effect) and the welfare gain or loss due to changes in the degree of insurance (insurance

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effect). For this reason, the social welfare function is widely used for incomplete market models with finitely lived consumers; for example, Conesa et al. (2009) use it to investigate the optimal capital income taxation. Formally, social welfare is defined as follows:

$$\mathbb{E}V = \sum_p \pi_p^0 \, V(1, p, 0) \tag{17}$$

where $V(.)$ is defined as in equation (1).

Second, I also look at cross-sectional welfare implications. Because of the rich heterogeneity of the model, it is also important to look at the heterogeneity of the welfare effects for different types of consumers. Specifically, I investigate the expected lifetime utility in the steady-state equilibrium for consumers with different initial productivity p. Since the productivity shock is highly persistent, looking at the welfare implications for consumers with different initial p roughly corresponds to studying the heterogeneous effects on consumers with different productivity potentials.

Finally, in Section 6, I will investigate the welfare effects associated with the rising indebtedness taking the equilibrium transition path between the 1970 and the 2000 economies into account. The analysis enables us to study the heterogeneous welfare effects on consumers in different cohorts along the transition path.

Notice that the important consideration in the current paper is the welfare loss due to the relaxed borrowing constraint and induced over-consumption. The preferences featuring temptation and self-control capture such a welfare loss naturally, because consumers succumb to the temptation of choosing consumption by discounting future value by $\beta\delta$, while the actual welfare is based on the discount factor δ. In other words, consumers choose consumption (savings) that is higher (lower) than the level associated with the highest welfare. The over-consumption (under-saving) problem is substantial. For example, if consumers have perfect self-control ($\gamma = 0$) in the model with $\beta = 0.70$, consumers make a consumption-savings decision based only on δ and the resulting capital-output ratio in the steady state is 22.5 percent higher than in the baseline temptation model.

Table 2 summarizes the welfare implications of rising indebtedness in the models with and without temptation. The three panels correspond to the no-temptation model, the baseline temptation model with $\beta = 0.70$, and the temptation model with a lower temptation discount factor ($\beta = 0.56$). For each model, two cases are shown. GE denotes the case where the 2000 steady-state economy is compared to the 1970 steady-state economy. PE denotes to the case where the prices are fixed at the 1970 levels, but the borrowing limit of the 2000 economy is used. The column marked as $\mathbb{E}V$ in Table 2 shows the changes in social welfare, which is defined as the ex-ante expected lifetime utility in the steady-state equilibrium, associated with the increased indebtedness. The changes are expressed as a percentage change in per-period consumption by moving from the 1970 economy to the 2000 economy. The comparison between the no-temptation model (the first panel) and the baseline temptation model (the second panel) shows that the welfare implications of a relaxed borrowing limit are very different, although, as shown in the previous section, the macroeconomic implications are similar. In the case where the prices are fixed at 1970 levels (partial equilibrium, or PE in Table 2), while the no-temptation model implies a welfare gain of 2.7 percent by moving from the 1970 economy (no borrowing) to the 2000

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Table 2: Welfare implications of rising indebtedness

		Welfare gain[1]		
		Initial productivity[3]		
GE[2]	$\mathbb{E}V$	Low	Medium	High
No-temptation model ($\beta = 1.00$)				
PE	+2.69	+6.63	+2.50	+0.16
GE	+0.65	+3.02	+0.52	+0.31
Temptation model ($\beta = 0.70$)				
PE	+0.61	+1.88	+0.18	+0.13
GE	−0.39	+0.44	−0.31	+0.96
Temptation model ($\beta = 0.56$)				
PE	−0.46	+0.81	−0.43	−0.61
GE	−1.09	−0.67	−1.13	+1.60

[1] Measured by the percentage increase in per-period consumption at all ages and states associated with the move from the 1970 steady-state economy (without debt) to the 2000 economy (with debt-to-output ratio of 7 percent).

[2] GE: general equilibrium. PE: partial equilibrium with prices fixed at the 1970 level.

[3] Low: consumers with the lowest initial p. Medium: consumers with the median initial p. High: consumers with highest initial p.

economy (with a debt-to-output ratio of 7 percent), the temptation model implies a substantially smaller welfare gain of 0.6 percent. The model with a higher degree of temptation (the third panel) implies a welfare *loss* of 0.5 percent. The general equilibrium effect offsets part of the gain or increases the loss, through lower output associated with lower capital stock. If the general equilibrium effect is taken into account, while the no-temptation model implies a welfare gain of 0.7 percent, the baseline temptation model implies a welfare *loss* of 0.4 percent. The temptation model with $\beta = 0.56$ implies an even larger welfare *loss* of 1.1 percent.

Figure 4 compares the heterogeneous welfare effects with and without a general equilibrium effect in the model without temptation. In the figure, each line represents the welfare gain of moving from the 1970 economy to the 2000 economy for consumers with different initial productivity p (1 is the lowest and 18 is the highest). The last three columns of Table 2 present welfare gains of consumers with low, medium, and high initial productivity. With respect to the welfare effects on consumers with different initial productivity, three groups with different initial productivity are affected differently in the model without temptation. First, those with low initial productivity benefit most from the relaxed borrowing limit. The consumers with the lowest initial productivity gain as much as 6.6 percent of per-period consumption in the partial equilibrium case, and 3.0 percent when the general equilibrium effect is considered. This is because the likelihood that they are constrained by the borrowing limit is highest for this group of consumers. However, they experience a welfare loss from the general equilibrium effect. In
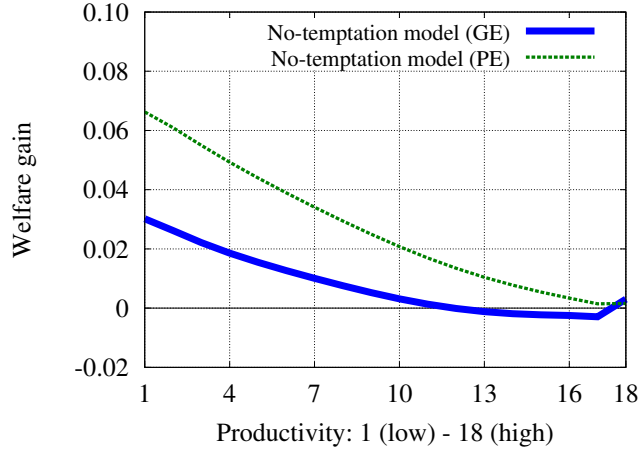
Figure 4: Heterogeneity of welfare gain in the no-temptation model: Partial and general equilibrium

Figure 4, the line representing the welfare effect with the general equilibrium effect considered, is located below the line representing the welfare effect without the general equilibrium effect, for consumers with low initial productivity. The reason is a lower equilibrium wage and a higher equilibrium interest rate, caused by a lower capital stock. Since the low-productivity consumers tend to borrow more often, and the main source of their income is labor income, both price effects hit the consumers negatively.

Second, the group with high initial productivity does not gain much from the relaxed borrowing limit. For those with the highest initial productivity in the no-temptation model, the welfare gain without the general equilibrium effect is a mere 0.16 percent increase in per-period consumption. Since it is not likely that they are constrained by the borrowing limit, they do not gain much from a relaxed borrowing limit. However, they gain from the general equilibrium effect. This is because they most likely remain savers throughout their lives, and they benefit from a higher interest rate, although part of the gain is offset by a lower wage. The welfare gain with the general equilibrium effect is equivalent to a 0.31 percent increase in per-period consumption. This contrasting general equilibrium effect for high and low productivity consumers is exactly what Campbell and Hercowitz (2009) emphasize in a different but closely related environment. In the model by Campbell and Hercowitz (2009) with secured credit, high discount rate consumers who remain borrowers lose from the general equilibrium effect, and low discount rate consumers gain from the general equilibrium effect. The general equilibrium effect is strong enough to incur a welfare loss for consumers with a high discount rate (consumers with low initial productivity in the current model) in their model, but the effect is not strong enough to overturn the welfare gain from the partial equilibrium effect here.

Finally, interestingly, consumers with medium initial productivity either enjoy a small welfare gain or suffer a welfare loss from the relaxed borrowing limit. In Figure 4, consumers with initial productivity of 12-17 in the no-temptation model suffer by moving from the 1970 economy to the 2000 economy. This is due to the combination of a weak welfare gain from the relaxed
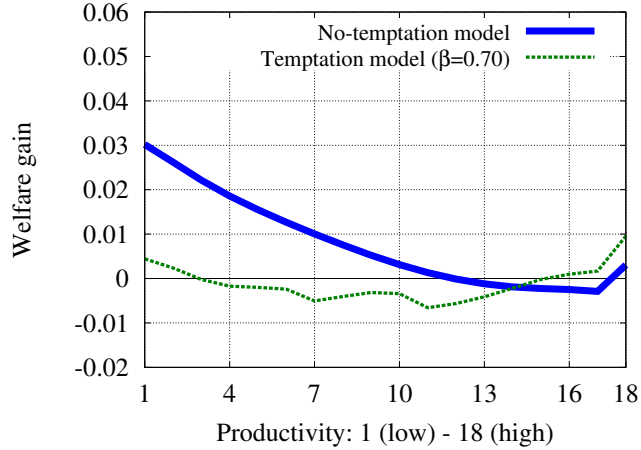
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Figure 5: Heterogeneity of welfare gain: Models with and without temptation

borrowing limit and a stronger negative welfare loss from a lower wage and a higher interest rate. As a result, the solid line in Figure 4, which represents the welfare effect for heterogeneous consumers, exhibits a U-shape. This is the same property that Obiols-Homs (2011) found in a similar environment.

Figure 5 compares the heterogeneity of the welfare effects across consumers with different initial productivity in the models with and without temptation. The general equilibrium effect is considered in the figure. Table 2 also contains the cross-sectional welfare effects for consumers with different initial productivity. Although the U-shape is also observed in the model with temptation in Figure 5, there are significant differences. The difference is especially striking for consumers with low initial productivity. Their gain from having a relaxed borrowing limit is significantly smaller in the case of the baseline temptation model. The key reason is the negative welfare effect of over-borrowing. Those who are close to the borrowing limit benefit from having a less strict borrowing limit, which facilitates consumption smoothing across ages and states, but suffer from borrowing more than the level associated with the highest welfare.

5.4 Optimal Borrowing Limit with Temptation

The discussion in the previous section implies that the *optimal* level of the borrowing limit differs, potentially substantially, among models with varying degree of temptation. Here I define the *optimal* borrowing limit as the level of the uniform borrowing limit that is associated with the highest social welfare defined as the ex-ante expected lifetime utility. Figure 6 exhibits social welfare, expressed as the increase in per-period consumption over the 1970 economy (the steady-state economy without debt), under different levels of the borrowing limit in the models with varying degree of temptation. The general equilibrium effect is taken into account. Three things are worth pointing out. First, the line for the model without temptation is located above the other lines, which are associated with the temptation models; the welfare gain is always higher in the no-temptation model, conditional on the same level of the borrowing limit. Second, all lines are hump shaped, because the negative general equilibrium effect from a lower capital stock
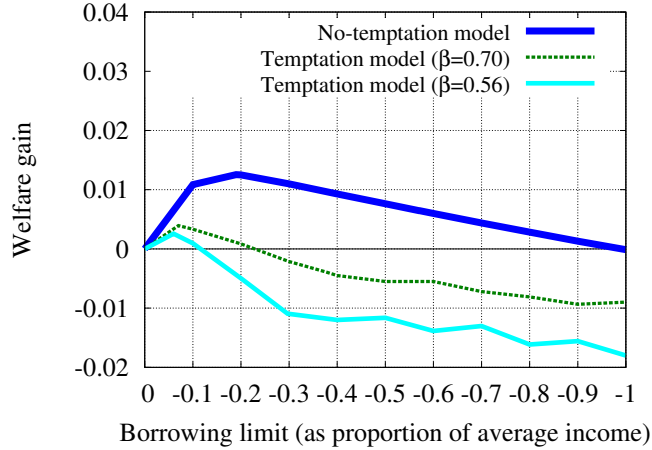
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Figure 6: Level of borrowing limit and social welfare

dominates at some point for all economies, as the borrowing limit becomes relaxed. Third, the optimal level of the borrowing limit, which is associated with the highest point of each line in Figure 6, becomes tighter in the degree of temptation. This is mainly because preferences featuring temptation and self-control imply a smaller (or negative) welfare gain from the relaxed borrowing limit for low and medium productivity consumers.

For the no-temptation model, the level of the uniform borrowing limit that maximizes social welfare is 19 percent of average income. This optimal level is lower than the level calibrated for the 2000 economy (57 percent). In other words, the model without temptation implies that the borrowing limit is too lax in the 2000 U.S. economy. At the optimal borrowing limit, the social welfare gain is 1.25 percent, which is close to the double of the welfare gain in the 2000 economy (0.65 percent). Cross-sectionally, consumers with the lowest initial productivity gain by 3.5 percent instead of 3.0 percent in terms of flow consumption if the economy is at the optimal borrowing limit instead of the 2000 economy. Consumers with medium productivity gain by 1.1 percent instead of 0.5 percent. On the other hand, the gain enjoyed by consumers with the highest productivity declines from 0.3 percent to 0.2 percent. In the baseline temptation model ($\beta = 0.70$), the optimal borrowing limit is 7 percent of average income. This optimal level is substantially lower than in the no-temptation model (19 percent) because of the welfare loss from over-borrowing. Furthermore, as in the no-temptation model, the optimal level is substantially lower than 37.6 percent, which is the borrowing limit corresponding to the 2000 economy. Indeed, the social welfare gain is positive (0.39 percent) if the optimal borrowing limit is implemented, compared to the welfare loss of 0.39 percent in the 2000 economy. In the case of the temptation model with a lower temptation discount factor ($\beta = 0.56$), the optimal borrowing limit declines further to 6 percent of average income. The social welfare gain is again positive (0.26 percent), compared to the 1.1 percent welfare loss in the 2000 economy.

In sum, when the general equilibrium effect is strong and causes social welfare to decline, implementing a tighter borrowing limit generates a welfare gain, in economies both with and without temptation. The difference between the models with and without temptation is that social wel-

fare starts to decline with a relatively tight borrowing limit in the temptation model, due to the over-consuming induced by the relaxed borrowing limit. Therefore, just as commitment by using an illiquid asset is valued in Laibson (1997) and forced saving might be welfare-improving in Malin (2008), tightening the borrowing limit in the temptation model can improve welfare as the tight borrowing limit prevents consumers from over-consuming, in addition to limiting the general equilibrium effect.

6   Results: Transition Analysis

This section presents the results of the analysis with the equilibrium transition path. In constructing the transition path between the initial steady state and the final one, I assume that the initial steady state is characterized by no borrowing ($\underline{a} = 0$). The initial steady state corresponds to the 1970 economy in Section 5. The final steady state is characterized by the borrowing limit associated with a debt-to-output ratio of 7 percent. This state corresponds to the 2000 economy in the previous section. Notice that the borrowing limit in the final steady state is different depending on the model, but all models generate the observed amount of debt in the 2000s. I assume that the borrowing limit relaxes linearly between period 0 (corresponds to 1970) and period 30 (corresponds to 2000).[17] After period 30, the borrowing limit stays at the level in the 2000 economy, while the economy converges to the 2000 steady state. In what follows, I first present the transition path of macroeconomic aggregates generated by the models (Section 6.1). The welfare analysis that explicitly takes into account the transition to the new steady state follows (Section 6.2).
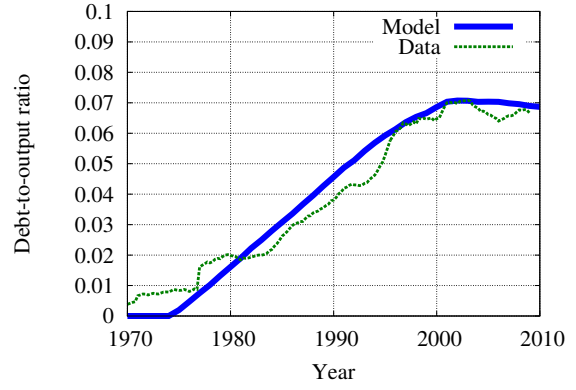
6.1   Macroeconomic Aggregates

Figure 7 compares the macroeconomic aggregates between 1970 and 2010, for the models with and without temptation. The results with the no-temptation model are on the left side, while those of the baseline temptation model ($\beta = 0.70$) are on the right side of the figure. Panels (a) and (b) compare the path of the debt-to-output ratio of the models and of the data (same as in Figure 1). It is clear that both models capture the dynamics of the debt-to-output ratio in the data quite well. In both models, the debt-to-output ratio gradually increases from the initial level of zero in 1970 and reaches about 7 percent around 2000. Panels (c) and (d) compare the transition path of the capital stock and output. Although there are some non-monotonic dynamics in the model with temptation, the long-run trend is a decline in the capital stock over time, as consumers borrow more or save less over time. As a result, output also continues to decrease over time. This is the source of the negative general equilibrium effect on welfare. Since labor is supplied inelastically, a declining capital stock yields a declining trend of wage and the increasing trend of the interest rate in the economy. These trends are present in both models, as shown in panels (e) and (f).

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[17] An alternative assumption is to let the borrowing limit jump to the level in the 2000 economy from the beginning of the transition (1971). However, it turns out that this alternative assumption generates a counterfactual transition path of the debt-to-output ratio: the debt-to-output ratio increases immediately in the 1970s, while the debt-to-output ratio gradually increases in the U.S. economy (Figure 1).
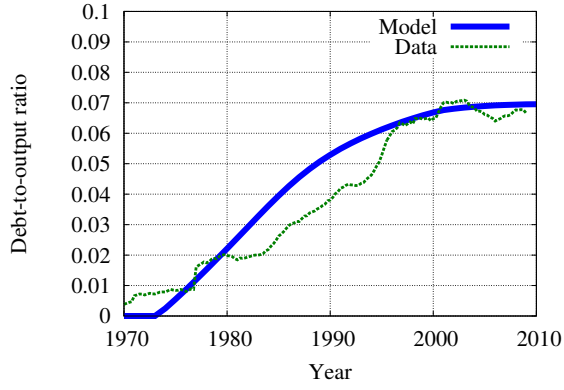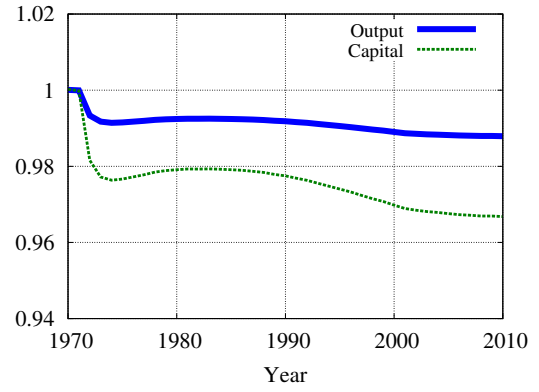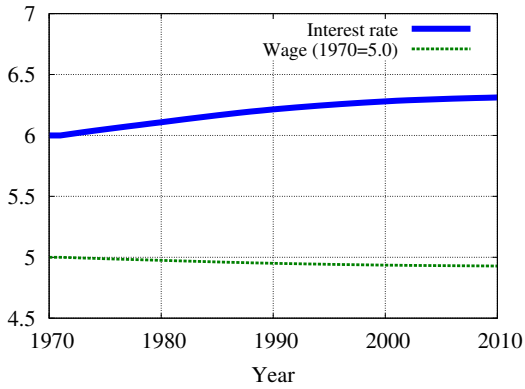
(a) No-temptation: debt/output
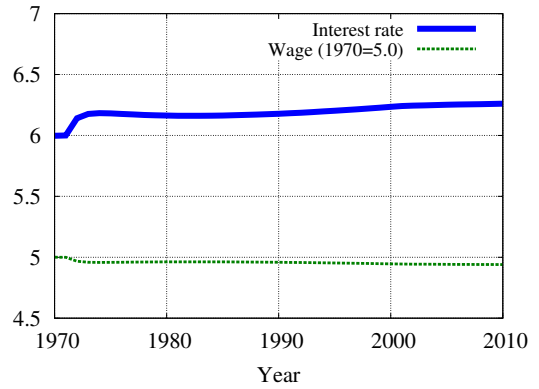
(b) Temptation: debt/output
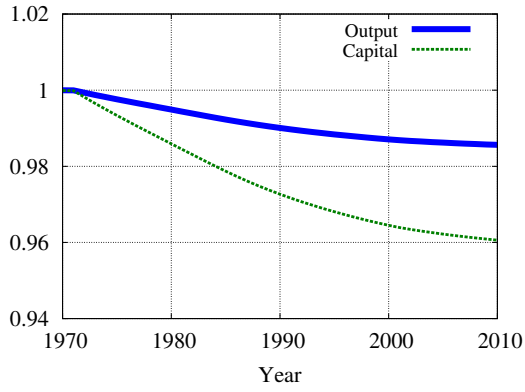
(c) No-temptation: capital and output

(d) Temptation: capital and output

(e) No-temptation: prices

(f) Temptation: prices

Figure 7: Comparison of macroeconomic aggregates along the transition path
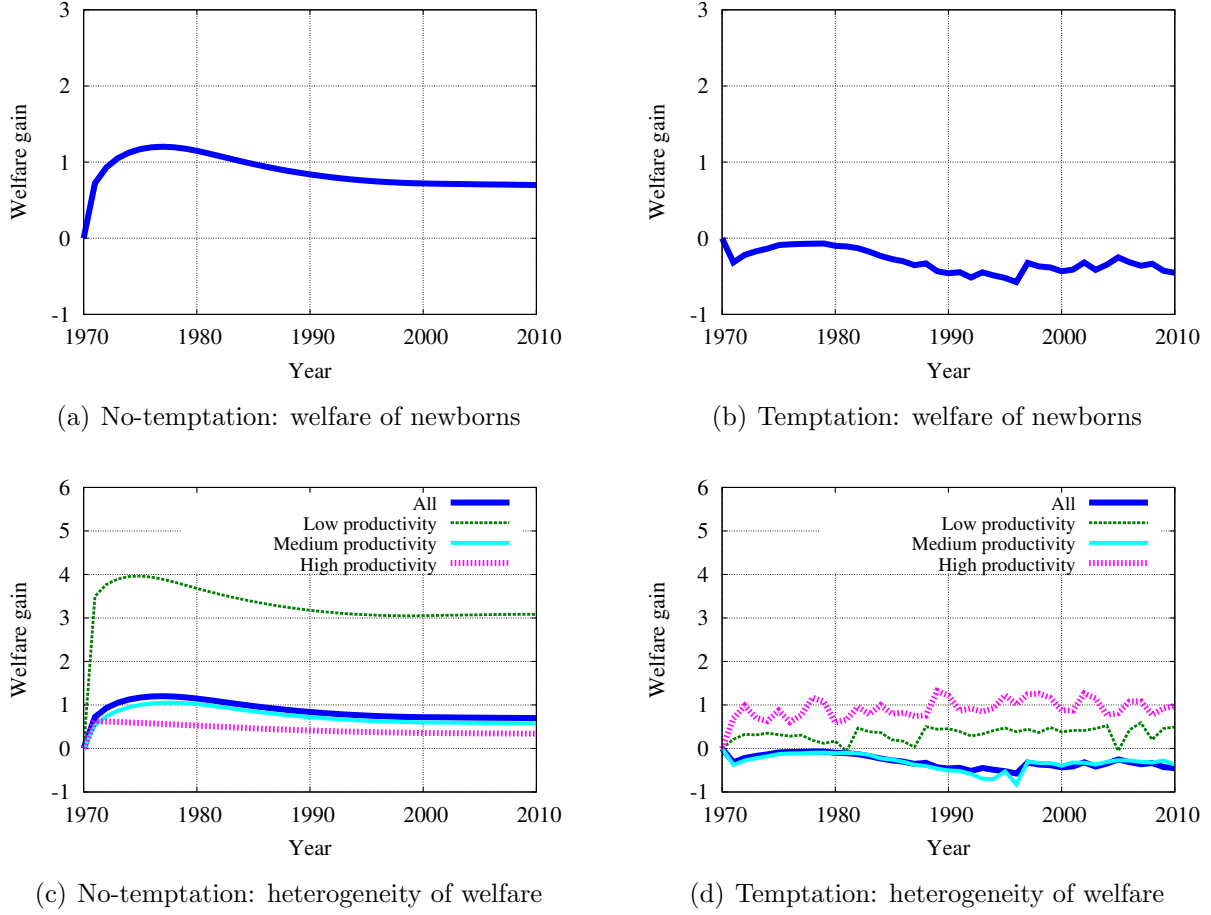
(a) No-temptation: welfare of newborns

(b) Temptation: welfare of newborns

(c) No-temptation: heterogeneity of welfare

(d) Temptation: heterogeneity of welfare

Figure 8: Comparison of welfare effects along the transition path

6.2 Welfare Implications

Similarly to what is shown in Section 5 using steady-state comparison, welfare implications are strikingly different between models with and without temptation along the transition path, although the dynamics of macroeconomic aggregates are similar. Figure 8 compares the welfare implications along the transition path in two models. Figures for the no-temptation model are on the left while those for the temptation model are on the right. First, panels (a) and (b) compare the ex-ante expected lifetime utility of newborns (age-20 consumers) in different years along the transition path. Welfare is measured as the uniform percentage increase in per-period consumption against the initial steady state. For example, in panel (a), the welfare gain is approximately 0.65 percent in 2000; this means that an age-20 consumer in 2000 along the transition path is better off than if he had been born in 1970 (the initial steady state), by an increase in per-period consumption equivalent to 0.65 percent. Three things can be learned from comparing panels (a) and (b). First, the welfare gain from a relaxed borrowing limit is substantially higher in the model without temptation throughout the transition path. Second, while the welfare effect is positive throughout the transition path in the no-temptation model,

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(a) No-temptation: proportion with welfare gain   (b) Temptation: proportion with welfare gain
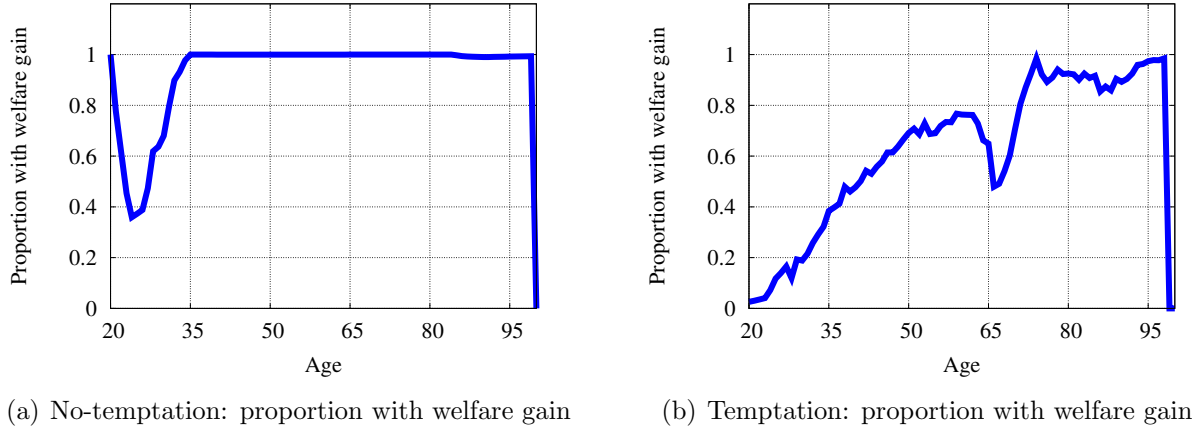
Figure 9: Comparison of welfare effects on the initial consumers

the welfare effect is negative for all generations in the temptation model. Third, in the model without temptation, the welfare effect is non-monotone; consumers born after 1975 are worse off than those born in 1975 as the general equilibrium effect associated with the decumulating capital stock intensifies along the transition path. However, the negative general equilibrium effect is quantitatively small compared with the large gain during the transition between 1970 and 2000 in the no-temptation model.

Panels (c) and (d) exhibit the heterogeneity of the welfare effects on newborns (age-20 consumers) with different initial productivity levels along the transition path in the models with and without temptation. The difference is striking. Consumers with low initial productivity gain substantially less in the temptation model. Whereas low-productivity consumers gain by 3-4 percent in the no-temptation model, the welfare gain is about 0.5 percent in the temptation model. In the temptation model, the relaxed borrowing constraint induces both a welfare gain (better consumption smoothing) and a welfare loss (over-consuming). Consumers with medium initial productivity, who basically determine the average welfare gain or loss of their cohort, gain by about 1 percent in the model without temptation, while they suffer a welfare loss throughout the transition path in the temptation model. Interestingly, those who gain the most along the transition path in the temptation model are the high-productivity consumers who gain mainly from the general equilibrium effect, while the high-productivity consumers gain the least in the no-temptation model.

In Figure 9, the proportion of consumers who gain from the transition to the 2000 economy among each age group in 1970 is shown, for both the model without temptation (panel (a)) and the temptation model (panel (b)). We can see that a very large proportion of consumers in 1970 gain from the switch to the transition path in the model without temptation. In total, 89 percent of consumers in 1970 (initial steady state) gain from the switch. On the other hand, in the temptation model, many consumers, especially the young ones, suffer from the transition. In total, less than half (49 percent) of consumers in 1970 gain from switching to the transition path.

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Table 3: Macroeconomic and welfare implications: sensitivity analysis

Economy[1]	GE[2]	$\underline{a}$[3]	D/Y	K[4]	Y[4]	r%	Var(c)[5]	$\mathbb{E}V$[6]
$\sigma = 3.0$, **no-temptation model**								
1970	–	0.000	0.000	1.000	1.000	6.00	0.575	–
2000	PE	−0.610	−0.079	0.917	0.969	6.00	0.548	+9.17
2000	GE	−0.610	−0.070	0.937	0.977	6.51	0.569	+5.31
$\sigma = 3.0$, **temptation model**								
1970	–	0.000	0.000	1.000	1.000	6.00	0.571	–
2000	PE	−0.470	−0.079	0.924	0.972	6.00	0.547	+5.30
2000	GE	−0.470	−0.070	0.940	0.978	6.49	0.566	+2.34
Endogenous labor, no-temptation model								
1970	–	0.000	0.000	1.000	1.000	6.00	0.472	–
2000	PE	−1.014	−0.077	0.928	0.972	6.00	0.470	+1.08
2000	GE	−1.014	−0.070	0.968	0.987	6.24	0.480	−1.31
Endogenous labor, temptation model								
1970	–	0.000	0.000	1.000	1.000	6.00	0.474	–
2000	PE	−0.815	−0.073	0.956	0.982	6.00	0.473	+0.52
2000	GE	−0.815	−0.070	0.977	0.991	6.16	0.480	−1.00

[1] 1970: Economy with no borrowing. 2000: Economy calibrated to debt-to-output ratio of 7 percent.

[2] GE: general equilibrium. PE: partial equilibrium with prices fixed at the 1970 level.

[3] Borrowing limit relative to total income.

[4] Level in the 1970 (no-debt) economy normalized to one.

[5] Cross-sectional variance of log-consumption, averaged across all age groups.

[6] Change in social welfare from the 1970 economy, measured as the percentage change in per-period consumption.

In sum, the transition analysis confirms the findings of the steady-state comparison: although the macroeconomic implications are similar between the models with different preference assumptions, the welfare implications are strikingly different if the welfare loss from over-borrowing is taken into account in the model with temptation and self-control. The ex-ante expected lifetime utility of newborns quickly rises above 1 percent (measured by per-period consumption growth) and slowly stabilizes at around 0.65 percent in 2000 in the no-temptation model, while the welfare effect stays negative and stabilizes at −0.4 percent in 2000 in the temptation model.

7 Sensitivity Analysis

This section provides sensitivity analysis with respect to the risk aversion parameter σ and endogenous labor supply. First, I implement the same steady-state analysis as in Section 5 with $\sigma = 3.0$ instead of the baseline value of $\sigma = 1.5$. The results are summarized in the top two panels of Table 3. As in the baseline experiments, the temptation model requires a more strict borrowing

limit (47 percent of average income) than in the model without temptation (61 percent). As in the baseline experiments, the macroeconomic implications of moving from the 1970 economy to the 2000 economy are similar between the models with and without temptation. On the other hand, the welfare gain of relaxing the borrowing limit is very different; it is equivalent to a 5.1 percent increase in flow consumption in the no-temptation model, while it is a 2.3 percent increase in the temptation model. Although both economies imply a social welfare gain from the relaxed borrowing limit as the gain from better consumption smoothing is larger with a higher risk aversion, the size of the welfare gain is very different between the two models.

The bottom two panels of Table 3 summarize the macroeconomic and welfare implications using the models with a labor-leisure decision. I assume the following non-separable utility function between consumption and leisure.

$$u(c, 1 - \ell) = \frac{(c^\mu (1 - \ell)^{1-\mu})^{1-\sigma}}{1 - \sigma} \tag{18}$$

I follow the same calibration strategy as for the baseline experiments, except for the new parameter μ. I calibrate μ such that the average time spent working is 33 percent of the disposable time (which is one). As for the partial equilibrium effect, as in the baseline results, the temptation model implies a lower welfare gain from the relaxed borrowing limit (equivalent to per-period consumption growth of 0.5 percent) than the model without temptation (1.1 percent). However, if the general equilibrium effect is taken into account, both models imply a welfare *loss* associated with increased indebtedness, and the loss is larger for the no-temptation model. Why? As studied by Pijoan-Mas (2006), with a labor-leisure decision, consumers can smooth consumption substantially through this channel. Therefore, the borrowing constraint is less important for consumption smoothing. This can be seen as the relatively small welfare gain in the partial equilibrium experiments. On the other hand, a weaker need for consumption smoothing means that the borrowing limit must be relaxed substantially to generate a debt-to-output ratio of 7 percent. Notice that the calibrated borrowing limit for the 2000 economy is very high for both models in Table 3. Besides, as in the baseline case, the no-temptation model implies a more lax borrowing limit. Under these circumstances, the stronger negative general equilibrium effect in the model without temptation dominates for the total welfare effect. This experiment implies that the endogenous labor supply decision, and the consumption smoothing through it, is crucially important in determining the welfare implications of a relaxed borrowing limit.

8 Conclusion

This paper investigates the macroeconomic and welfare implications of rising indebtedness in the U.S. using the model with preferences featuring temptation and self-control. The temptation model can capture the welfare loss associated with over-borrowing, together with the gain from better consumption smoothing and the general equilibrium effect. There are three main findings. First, not only are the models with and without temptation observationally similar in terms of macroeconomic aggregates in the steady state, but they also have similar predictions in terms of macroeconomic response to a relaxed borrowing limit. Second, although the macroeconomic implications of the relaxed borrowing limit are similar between the two models, the welfare implications are very different; the temptation models imply significantly lower or even negative

welfare effects associated with rising indebtedness. In particular, I find that when debt increases to the same extent as in the period 1970-2000, there is a loss of social welfare equivalent to a 0.39 percent decrease in per-period consumption in the temptation model. On the other hand, the standard model without temptation implies a social welfare gain equivalent to a 0.65 percent increase in per-period consumption. Finally, the optimal borrowing limit becomes tighter when the degree of temptation becomes higher. A tight restriction on borrowing could be welfare-improving according to the temptation model, not only because it weakens the negative general equilibrium effect, but also it helps consumers avoiding over-borrowing.

Even though the models with and without temptation are observationally similar along many dimensions, they have very different welfare implications. Therefore, from the normative perspective, it is important to find other and better ways to distinguish between the two models, although there might be little need to use the non-standard preferences for a positive analysis. I list two potential ways to distinguish. First, if we can observe the borrowing constraint, consumers' response to changes in the borrowing constraint can be used to identify the degree of temptation. It would be even more helpful if the borrowing constraint for each individual consumer can be observed. Second, it might be possible to combine the model implications and survey data to distinguish between the two models, if the survey data can be mapped into welfare implications. This issue is left for future research.

One interesting and important extension from the current paper is associated with consumer bankruptcy. The increase in consumer debt has been accompanied by a substantial increase in consumer bankruptcy filings. White (2007) argues that a high level of consumer bankruptcies can be better understood using hyperbolic discounting/temptation preferences. Recently, the consumer bankruptcy law was reformed to make bankruptcy more costly and not available to consumers with relatively high incomes in order to discourage abuse of the law. The standard equilibrium models of consumer bankruptcy imply that a tougher bankruptcy law would benefit consumers by allowing a stronger commitment to repay. But it is not clear if the intuition carries over when consumers suffer from over-borrowing. Nakajima (2009) investigates this issue.

References

Aiyagari, S. Rao, "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 1994, *109* (3), 659–684.

Angeletos, George-Marios, David Laibson, Andrea Rupetto, Jeremy Tobacman, and Stephen Weinberg, "The Hyperbolic Consumption Model: Calibration, Simulation, and Empirical Evaluation," *Journal of Economic Perspectives*, 2001, *15* (3), 47–68.

Barro, Robert J., "Ramsey Meets Laibson in the Neoclassical Growth Model," *Quarterly Journal of Economics*, 1999, *114* (4), 1125–1152.

Budría, S., Javier Díaz-Gimenez, Vincenzo Quadrini, and Jose-Víctor Ríos-Rull, "Updated Facts on the U.S. Distributions of Earnings, Income and Wealth," *Federal Reserve Bank of Minneapolis Quarterly Review*, 2002, *26* (3), 2–35.

Campbell, Jeffrey R. and Zvi Hercowitz, "Welfare Implications of the Transition to High Household Debt," *Journal of Monetary Economics*, 2009, *56* (1), 1–16.

Castañeda, Anna, Javier Díaz-Giménez, and José-Víctor Ríos-Rull, "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 2003, *111* (4), 818–857.

Conesa, Juan Carlos, Sagiri Kitao, and Dirk Krueger, "Taxing Capital? Not a Bad Idea After All!," *American Economic Review*, 2009, *99* (1), 25–48.

Federal Reserve Bank of Dallas, "Annual Report: By Our Own Bootstraps, Economic Opportunity and the Dynamics of Income Distribution," 1995.

Gul, Faruk and Wolfgang Pesendorfer, "Temptation and Self-Control," *Econometrica*, 2001, *69* (6), 1403–1435.

_ **and** _ , "Self-Control and the Theory of Consumption," *Econometrica*, 2004, *72* (1), 119–158.

_ **and** _ , "Self-Control, Revealed Preference and Consumption Choice," *Review of Economic Dynamics*, 2004, *7* (2), 243–264.

Huggett, Mark, "Wealth Distribution in Life-Cycle Economies," *Journal of Monetary Economics*, 1996, *38* (3), 469–494.

İmrohoroğlu, Ayşe, Selahattin İmrohoroğlu, and Douglas H. Joines, "Time-Inconsistent Preferences and Social Security," *Quarterly Journal of Economics*, 2003, *118* (2), 745–784.

Krusell, Per, Burhanettin Kuruşcu, and Anthony A. Smith, "Temptation and Taxation," *Econometrica*, 2010, *78* (6), 2063–2084.

Laibson, David, "Hyperbolic Discount Functions, Undersaving and Savings Policy," *National Bureau of Economic Research Working Paper*, 1996, *No. 5635*.

_ , "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics*, 1997, *112* (2), 443–477.

_ , **Andrea Repetto, and Jeremy Tobacman**, "A Debt Puzzle," in Philippe Aghion, Roman Frydman, Joseph Stiglitz, and Michael Woodford, eds., *Knowledge, Information, and Expectations in Modern Economics: In Honor of Edmund S. Phelps*, Princeton: Princeton University Press, 2003, chapter 11, pp. 228–166.

_ , _ , **and** _ , "Estimating Discount Functions with Consumption Choices over the Lifecycle," *National Bureau of Economic Research Working Paper*, 2007, *No. 13314*.

Livshits, Igor, James MacGee, and Michele Tertilt, "Accounting for the Rise in Consumer Bankruptcies," *American Economic Journal: Macroeconomics*, 2010, *2* (2), 165–193.

Malin, Benjamin A., "Hyperbolic Discounting and Uniform Savings Floors," *Journal of Public Economics*, 2008, *92* (10-11), 1986–2002.

McGrattan, Ellen R. and Richard Rogerson, "Changes in Hours Worked, 1950-2000," *Federal Reserve Bank of Minneapolis Quarterly Review*, 2004, *28*, 14–33.

Nakajima, Makoto, "Equiribrium Default and Temptation," 2009. Unpublished.

Obiols-Homs, Francesc, "On Borrowing Limits and Welfare," *Review of Economic Dynamics*, 2011, *14* (2), 279–294.

Phelps, Edmund S. and Robert A. Pollak, "On Second-Best National Saving and Game-Equilibrium Growth," *Review of Economic Studies*, 1968, *35* (2), 185–199.

Pijoan-Mas, Josep, "Precautionary Savings or Working Longer Hours?," *Review of Economic Dynamics*, 2006, *9* (2), 326–352.

Smith, Anthony A., "Comment on: Welfare Implications of the Transition to High Household Debt by Campbell and Hercowitz," *Journal of Monetary Economics*, 2009, *56* (1), 17–19.

Social Security Administration, *Annual Statistical Supplement, 2006, to the Social Security Bulletin*, Washington, DC: Social Security Administration, 2007.

Storesletten, Kjetil, Chris I. Telmer, and Amir Yaron, "Consumption and Risk Sharing over the Life Cycle," *Journal of Monetary Economics*, 2004, *51* (3), 609–633.

Strotz, Robert H., "Myopia and Inconsistency in Dynamic Utility Maximization," *Review of Economic Studies*, 1956, *23* (3), 165–180.

Tauchen, George, "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics Letters*, 1986, *20* (2), 177–181.

Tobacman, Jeremy, "Endogenous Effective Discounting, Credit Constraints, and Wealth Inequality," *American Economic Review*, 2009, *99* (2), 369–373.

White, Michelle J., "Bankruptcy Reform and Credit Cards," *Journal of Economic Perspectives*, 2007, *21* (4), 175–200.

Supplement to "Rising Indebtedness and Temptation: A Welfare Analysis": Appendices[*]

Makoto Nakajima[†]

December 2, 2011

Appendix A: Additional Details of Calibration

Figure 1 shows the age-specific survival probability, which is used as $\{s_i\}_{i=1}^I$ in the calibration. Figure 2 compares the discount factors of the standard exponential discounting and hyperbolic discounting models for periods (years) 1 to 50. The calibrated β and δ are used. The figure shows that the discount factor drops substantially more from period 1 to 2 in the case of the hyperbolic discounting preferences. On the other hand, the discount factor applied to utility in the distant future is higher for the hyperbolic discounting model. Laibson (1997) argues that housing, from which inhabitants can enjoy utility as long as they own it and live in it, has an extra value for hyperbolic discounting consumers, since the dividends can be enjoyed for a long period of time. Figure 3 shows the average life-cycle profile of labor productivity. This is used as $\{e_i\}_{i=1}^I$ in the calibration.

Appendix B: Computation Algorithm

I will first describe below the computational algorithm to solve the steady-state equilibrium of the model with temptation. Since the focus is the steady-state equilibrium, I drop the time script in the algorithm. The solution method for the model without temptation (i.e., exponential discounting model) is straightforward and thus omitted. Adding the labor-leisure decision is also straightforward.

Algorithm 1 (Computation algorithm for solving steady-state equilibrium)

1. *Set the initial guess of the aggregate capital stock K^0 and the per-consumer transfer d^0. Notice that the aggregate labor supply $\overline{L}$ can be computed independently from the model since there is no labor supply decision.*

2. *Given K^0 and $\overline{L}$, compute the interest rate r and the wage w. The transfer used in the iteration is equal to the guess, i.e., $d = d^0$. The Social Security benefits $\overline{b}$ can be computed*
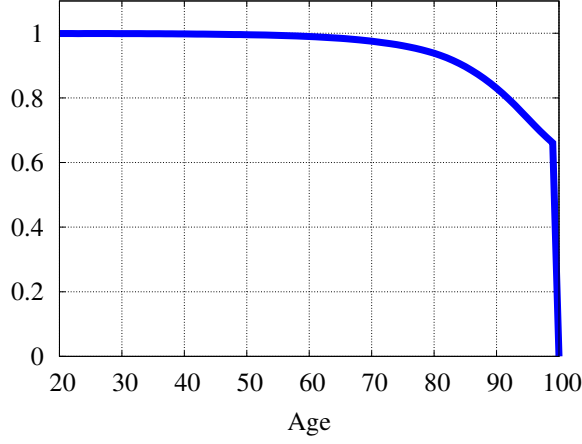

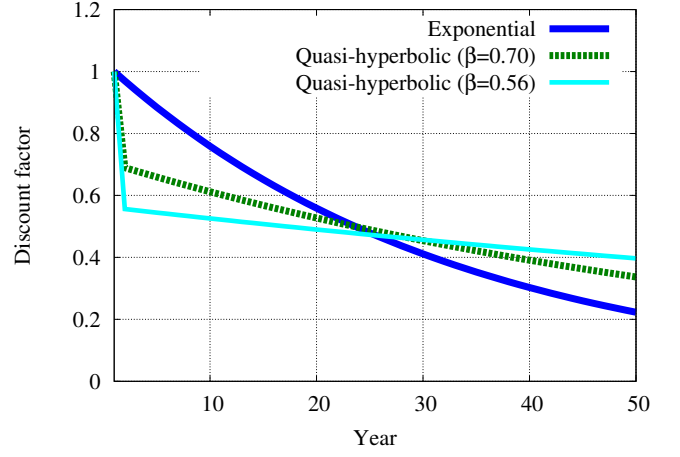Figure 1: Age-specific survival probability

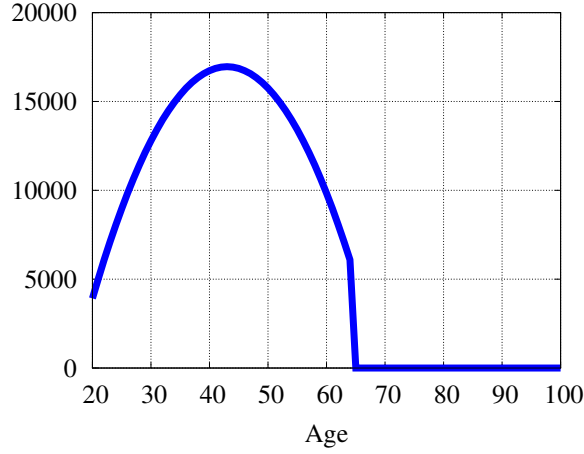Figure 2: Comparison of discount factors

Figure 3: Average life-cycle profile of labor productivity

using the government budget constraint (9) together with w and the type distribution of consumers with respect to age and labor productivity. Once $\bar{b}$ is obtained, $\{b_i\}_{i=1}^{I}$ is set as $b_i = \bar{b}$ for $i \geq I_R$ and $b_i = 0$ for $i < I_R$.

3. Given $\{r, w, d, \{b_i\}_{i=1}^{I}\}$, solve the consumer's optimization problem using backward induction.

   (a) Set $V(I+1, p, a) = 0$ for all p and a.

   (b) Solve the problem of the age-I consumer, using the Bellman equation (1) for all (p, a). The optimal level a' is obtained by basically comparing values conditional on a' and choosing a' associated with the highest value. Notice that, since $\gamma \to \infty$, the optimal a' is chosen to maximize the temptation utility, while the value function is updated

using the self-control utility.

   (c) With $V(I, p, a)$ at hand, we can solve the problem of the age-$I-1$ consumer. Keep going back in the same way until the value function and the optimal decision rule for the age-1 (initial age) consumer are obtained.

4. *Using the obtained optimal decision rule $g^a(i, p, a)$, simulate the model.*

   (a) Set the type distribution for the newborns ($i = 1$), which is exogenously given. In particular, all newborns have $i = 1$ and $a = 0$. Initial p is distributed according to π_p^0.

   (b) Update the type distribution using the stochastic process for p and the optimal decision rule $g^a(i, p, a)$, and obtain the type distribution for $i = 2$.

   (c) Keep updating until $i = I$ (last age).

5. *Compute the aggregate capital stock K^1 and the total amount of the accidental bequests implied by the simulated distribution. Notice that consumers survive according to the age-dependent survival probability, and there is population growth, which makes the size of the younger population larger. Make these adjustments when computing the aggregate capital stock and the total amount of the accidental bequests. Specifically, when the measure of age-1 consumers is normalized to one, the measure of age-i consumers, $\widetilde{\mu}_i$, can be represented as follows:*

$$\widetilde{\mu}_i = \frac{1}{(1+\nu)^{i-1}} \prod_{j=0}^{i-1} s_j$$

where $s_0 = 1$. Once the aggregate amount of the accidental bequests is computed, we can compute the per-consumer lump-sum transfer d^1 using the government budget constraint (10).

6. *Compare $\{K^0, d^0\}$ and $\{K^1, d^1\}$. If they are closer than the predetermined tolerance level, stop. Otherwise, update $\{K^0, d^0\}$ and go back to step 2.*

Next, I will describe the solution algorithm of an equilibrium that features the deterministic transition between two steady states. The first step is to obtain the two steady states using Algorithm 1. Denote the initial and the new steady state by $t = 0$ and $t = \infty$, respectively. Set the initial distribution along the transition path μ_0 as the type distribution of consumers in the initial steady state, and the value at the end of the transition $V_\infty(i, p, a)$ as the value function in the new steady state. The only difference between the two steady states is the borrowing limit $\underline{a}$; total factor productivity Z is assumed to be constant over time. I also assume that the transition is complete after $T < \infty$ periods. Since the model economy converges to the new steady state only asymptotically, a large T is desirable for a good approximation. Now, in period 0 the economy is in the initial steady state, but in period 1, the transition, in particular the sequence of the borrowing limit $\{\underline{a}_t\}_{t=1}^T$, is revealed to consumers. Let $\underline{a}_1 = \underline{a}_0 = 0$, $\underline{a}_t = \underline{a}_\infty$ (the borrowing limit in the 2000 economy) for $t = \widetilde{T}, \widetilde{T} + 1, \widetilde{T} + 2, ..., T$, and $\underline{a}_t$ gradually increases

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between period 1 and period $\widetilde{T} < T$. I set $\widetilde{T} = 30$. Since $t = 0$ corresponds to 1970 (initial steady state without borrowing) and one period is a year, $t = \widetilde{T} = 30$ corresponds to 2000. After 2000, the borrowing limit is assumed to remain at the same level as in 2000.

Algorithm 2 (Computation algorithm for solving equilibrium transition path)

1. *Set the initial guess of the sequence $\{K_t^0, d_t^0\}_{t=0}^{T}$. Notice that the sequence of aggregate labor supply $\{\overline{L}_t\}_{t=0}^{T}$ can be computed independently from the model.*

2. *Given $\{K_t^0, d_t^0, \overline{L}_t\}_{t=0}^{T}$, compute the sequence $\{r_t, w_t, d_t, \{b_{t,i}\}_{i=1}^{I}\}_{t=0}^{T}$.*

3. *Given $\{r_t, w_t, d_t, \{b_{t,i}\}_{i=1}^{I}\}_{t=0}^{T}$ and $\{\underline{a}_t\}_{t=0}^{T}$, solve the consumer's optimization problem using backward induction.*

   (a) *Start from period T. Notice that we know the value function $V_{T+1}(i, p, a) = V_\infty(i, p, a)$ for $\forall(i, p, a)$ since the economy is assumed to have converged to the new steady state in period T.*

   (b) *Solve the consumer's problem for $\forall(i, p, a)$ in period T, given $V_{T+1}(i, p, a)$. The solution method for the model with temptation is the same as in the steady-state equilibrium described in Algorithm 1. The optimal decision rule in period T, $g_T^a(i, p, a)$, and the value function for period T, $V_T(i, p, a)$, are obtained. Notice that since the value function for period $T + 1$ is known, there is no need to go back from age I as in Algorithm 1.*

   (c) *Keep going back until $t = 0$.*

4. *Using the obtained sequence of optimal decision rules $\{g_t^a(i, p, a)\}_{t=0}^{T}$, simulate the model.*

   (a) *The type distribution in period 0 is given by μ_0.*

   (b) *Update the type distribution using the stochastic process for p and the optimal decision rule for period t, $g_t^a(i, p, a)$, and obtain the type distribution in period 1 (μ_1). Make sure to normalize the population size each period.*

   (c) *Keep updating until period T (last period).*

5. *Compute $\{K_t^1, d_t^1\}_{t=0}^{T}$ using the sequence of type distribution $\{\mu_t\}_{t=0}^{T}$ generated in the last step.*

6. *Compare $\{K_t^0, d_t^0\}_{t=0}^{T}$ and $\{K_t^1, d_t^1\}_{t=0}^{T}$. If they are closer than the predetermined tolerance level, stop. Otherwise, update $\{K_t^0, d_t^0\}_{t=0}^{T}$ and go back to step 2.*

References

Laibson, David, "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics*, 1997, *112* (2), 443–477.