

## 7. Default and Aggregate Fluctuations in Storage Economies

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**Abstract:** In this paper we extend the work of Chatterjee, Corbae, Nakajima and Ríos-Rull (unpublished manuscript, University of Pennsylvania, 2002) to include aggregate real shocks to economic activity. The model, which includes agents that borrow and lend and a competitive credit industry, and which has endogenous default and credit limits, allows us to explore the extent to which aggregate events are amplified or smoothed via the mechanism of household bankruptcy filings. In the model agents are subject to shocks to earnings opportunities, to preferences, and to their asset position and borrow and lend to smooth consumption. On occasion, the realization of the shocks is bad enough so that agents take advantage of the opportunities provided by the U.S. Bankruptcy Code and file for bankruptcy, which wipes out their debt at the expense both of being banned from borrowing for a certain amount of time and of incurring transaction costs. The incentives to default are time-varying and depend on the individual state and general economic conditions. The model is quantitative in the sense that its fundamental parameters are estimated using U.S. data, and the model can replicate the aggregate conditions of the U.S. economy. Especially, the model accounts for the very high number of bankruptcies in the past few years. We report statistics produced by experiments with model economies with various aggregate shocks. Based on these experiments, we analyze the reaction of households to various aggregate real shocks and the interaction between households and the credit industry, and we discuss the aggregate implications of these actions and the direction in which the model might be further extended.

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<sup>1</sup>Ríos-Rull thanks the National Science Foundation, the University of Pennsylvania Research Foundation, the Spanish Ministry of Education, and the Centro de Altísimos Estudios Ríos Pérez. We thank the organizers of the conference to honor Herbert Scarf.

## 7.1 Introduction

In this chapter we study a model economy where agents file for bankruptcy that is mapped quantitatively to the U.S. economy and that is subject to aggregate real shocks. We extend the work of Chatterjee et al. (2002) (who studied the steady state of economies with bankruptcy where equilibrium interest rates are indexed by a set of individual characteristics) to study aggregate uncertainty. The aggregate shocks that we study generate expansions and recessions in a variety of ways: (i) a good realization shifts the distribution of efficiency units of labor to the right, and a bad realization shifts it to the left; (ii) shocks increase or decrease the risk-free interest rate; (iii) shocks affect the number of people who suffer asset destruction; or (iv) all of the shocks above occur in combination.

The current chapter is part of an ongoing investigation into the role of bankruptcy filings and general credit disruptions in shaping business cycles. In general the study of these issues is very hard: not only does it include many agents differing in asset holdings, shocks, and credit ratings, implying that the state of the economy is a probability measure over these characteristics, but also solving the individual problem requires the forecasting of prices, which in turn requires the whole state vector to be part of the individual state. To get around this problem we follow the approach used in Diaz-Gimenez et al. (1992), which specifies the model in a certain way, so that prices are essentially exogenous and hence do not depend on the distribution of agents over states. We are able to do so by (i) assuming a storage technology with exogenous rate of return and by (ii) effectively preventing loans from being held at the time of the resolution of aggregate variables. Only idiosyncratic shocks are realized while loans are outstanding, which guarantees that the law of large numbers applies and that firms' profits coincide with the expected profits (which are zero) in all states of nature, guaranteeing that prices can be forecast based solely on aggregate exogenous variables.

Although the timing that we choose guarantees that the distribution of agents does not affect prices and hence that the model can be solved with relative ease,<sup>2</sup> it also takes away part of the properties that we are interested in: the possible transmission of credit crunches throughout the economy as surprise increases in default will in turn induce more

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<sup>2</sup>The estimation stage of the model still requires us to use a Beowulf cluster.

defaults. To address this issue, a different approach is needed, summarizing the distribution of household types by some of its statistics as in Krusell and Smith (1998) and especially Krusell and Smith (1997) (this is the subject of our research agenda starting with Nakajima and Ríos-Rull (2003)).

We estimate the fundamental parameters of the model using U.S. data so that the model can replicate the aggregate conditions of the U.S. economy. Especially, the model accounts for the very high number of bankruptcies in the past few years. Using the calibrated model, we analyze the interaction between various aggregate real shocks and household behavior on bankruptcy filings.

Section 7.2 lays out the model. Section 7.3 specifies a parameterization of the model that has a steady state that can be mapped to the U.S. data. Section 7.4 discusses the experiments that we run. They involve different specifications of what business cycles are but they are all chosen to generate aggregate business cycles statistics like those in the data. Section 7.5 describes the business cycle properties of the baseline model economy, and Section 7.6 studies those properties in the rest of the model economies that we are interested in. Section 7.7 concludes.

## **7.2 The Model Economy**

The model is a version of Chatterjee et al. (2002) with aggregate shocks. In the model, agents are subject to idiosyncratic and aggregate shocks, which affect their assessment of consumption and their availability of resources. There are no markets to insure against these contingencies. Moreover, the market structure resembles that of the U.S., where households can borrow and save. They save according to a given storage technology or world interest rate, whereas they can borrow at market interest rates that reflect the fact that households can file for bankruptcy, which condones their debts. This option, which households may use unilaterally, inflicts minimal transaction costs on them and prevents them from having access to future credit for a certain number of years. Aggregate shocks affect the properties of the idiosyncratic earnings shocks.

As we stated above, in this project we specify the aggregate shocks in such a way that they do not generate uncertainty in the realized profits of firms, ensuring that prices of

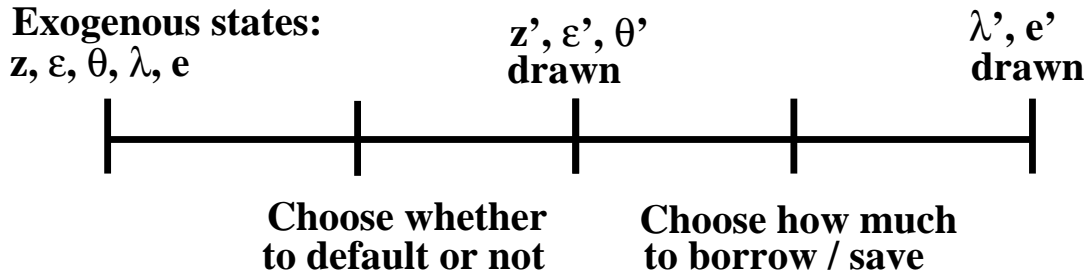


Figure 7.1: Timing of events within a period.

loans, although depending on both the aggregate shocks and the specific circumstances of the borrower, do not depend on the whole distribution of agents. We achieve this by posing a particular timing in the model so that aggregate uncertainty does not affect the default decisions of households. Only idiosyncratic shocks occur while agents hold loans, but the rate of return of these loans is perfectly forecastable. We now proceed to describe the timing of the model.

### 7.2.1 The Timing

At the beginning of each period, the economy is in an aggregate exogenous state  $z \in Z$  that follows a Markov process with transition  $\Pi_{z,z'}$ , (we use the standard notation of using  $z, z'$  to refer to the current and the following periods' value of the variable). Individual households are characterized by a vector of exogenous stochastic characteristics that take only finitely many values and by a value of earnings. These characteristics include a shock that governs the evolution of earnings,  $\varepsilon \in \{\varepsilon_1, \dots, \varepsilon_{n_\varepsilon}\} = \mathcal{E}$ , a shock that affects the utility of consumption, which we denote as  $\theta \in \{\theta_1, \dots, \theta_{n_\theta}\} = \Theta$ , and a shock that affects asset destruction,  $\lambda \in \{\lambda_1, \dots, \lambda_{n_\lambda}\} = \Lambda$ . The shock that affects earnings,  $\varepsilon$ , has the property of affecting the probability distribution from which households draw their actual earnings. Earnings  $e$  has a continuous domain, i.e.,  $0 < e \in [e, \bar{e}] = E$  and has a continuous c.d.f. given by  $F(e, \varepsilon, z)$ , where  $\varepsilon$  and  $z$  are affecting the actual probability that earnings are less than or equal to  $e$ . We denote the vector of individual shocks by  $s = \{\varepsilon, \theta, \lambda, e\} \in S$ . The timing of the realizations of the shocks is given by Fig. 7.1, which also includes the timing of the decisions that households make.

The individual shocks  $\{\varepsilon, \theta, \lambda\}$  follow Markov processes that are also conditional on the aggregate state. In this fashion, we write the transition matrices of the Markov processes as  $\pi_{\theta'|z, \theta, z'}$ ,  $\pi_{\varepsilon'|z, \varepsilon, z'}$ , and  $\pi_{\lambda'|z, \lambda, z'}$ . We write the joint Markov chain that yields the evolution of the individual state variables by  $\pi_{s'|z, s, z'}$ , although they are not updated simultaneously. We denote the joint transition matrix of individual and aggregate shocks by  $\Gamma_{z, s, z' s'}$ . Note that we allow the individual transitions to depend on the aggregate shocks of two consecutive periods. This is to give ourselves the possibility of having the aggregate measure of people depend only on the aggregate shock and not on the whole history (see Castañeda et al. (1998) for details).

We also use the notation  $\tilde{s} = \{\varepsilon', \theta', \lambda\} \in \tilde{S}$  to denote the individual state at the time of the saving and borrowing decision, excluding current earnings  $e$  (note that  $e$  carries no predictive power over tomorrow's earnings, which depend only on  $z'$  and  $\varepsilon'$ ), and having the value of the shock to asset holdings,  $\lambda$ , at its previous period value. Associated with this notation we have  $\tilde{\Gamma}_{z, s, z', \tilde{s}}$ .

A crucial step in the development of the model is the identification of which variables index prices. We proceed by guessing which variables these are and then establishing that indeed these variables are sufficient to characterize prices.

### 7.2.2 The Default Option and Market Arrangement

We model the default option to be like a filing for bankruptcy under the U.S. Bankruptcy Code. Let  $h \in \{0, 1\}$  denote the “bankruptcy flag” for a household, where  $h = 1$  indicates a record of a bankruptcy filing in the household's credit history and  $h = 0$  denotes the absence of any such record. We can interpret  $h$  as the household's credit rating, which is either good ( $h = 0$ , not having filed for bankruptcy recently) or bad ( $h = 1$ , having filed for bankruptcy recently and U.S. law allowing this information to be public). Consider a household that starts the current period with a good credit rating and some unsecured debt. If the household files for bankruptcy (and we permit a household to do so irrespective of its current income or past consumption level), the following things happen:

1. The household's liabilities are set to zero (i.e., its debts are discharged) and the household is not permitted to save in the current period. The latter assumption is

a simple way to recognize that a household's attempt to accumulate assets during the filing period will result in those assets being seized by creditors.

2. The household begins the next period with a bad credit rating (i.e.,  $h' = 1$ ).
3. A household whose beginning-of-period credit rating is bad (i.e.,  $h = 1$ ) cannot get any new loans.<sup>3</sup> Also, a household with a bad credit rating experiences a loss equal to a fraction  $0 < \gamma < 1$  of earnings, a loss intended to capture the pecuniary costs of a bad credit rating.
4. A household with a bad credit rating will keep its bad credit rating in the following period with exogenous positive probability  $\eta$  and will recover a good credit rating with probability  $(1 - \eta)$ . This is a simple, albeit idealized, way of modeling the fact that a bankruptcy flag remains on an individual's credit history for only a finite number of years.

The addition of the default option implies that profit-maximizing lenders take into consideration the probabilities of default of the borrowers. Different types of borrowers have different default probabilities, which will imply that the loans are indexed by whatever characteristics of the borrower may affect those probabilities.

We restrict households to choosing from a menu of loans, which we model as a single one-period pure discount bond with a face value in a finite set  $L^-$  that has only elements with negative values.<sup>4</sup> A purchase of a discount bond with a negative face value  $\ell'$  means

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<sup>3</sup>This feature requires some discussion. Filing for bankruptcy implies giving up the right to file again for seven years. Then why does a bad credit rating imply that households cannot borrow anymore? We can rationalize the inability to borrow in two ways. One is by modeling the environment as a game and showing that there exists an equilibrium with the characteristic that nobody borrows or lends. This type of equilibrium arises because this game is a coordination game. The other rationalization, which we prefer, is via regulation. Public overseers of private lenders do not approve of loans to people with a bad credit rating. This is just a matter of policy. Note also that private lenders, once they have outstanding loans, want this regulation in place because it gives a rationale for borrowers to try to avoid defaulting by imposing a penalty when they do so.

<sup>4</sup>The finiteness of the set of possible loans is assumed just to have a finite set of prices that simplifies the analysis. Quantitatively, this poses no real restrictions, because the step between consecutive loan sizes can be made arbitrarily small.

that the household has entered into a contract where it promises to deliver, conditional on not declaring bankruptcy,  $-\ell' > 0$  units of the consumption good next period; if it declares bankruptcy, the household delivers nothing.

We conjecture that the price of a loan of size  $\ell'$ , which is the per-unit amount of goods that a type  $\tilde{s}$  in aggregate state  $z'$  gets in exchange for a liability next period of size  $\ell'$ , is only a function of these variables and not of any distributional variable or of any time subscript. We denote this price by  $q_{z',\tilde{s},\ell'} \geq 0$ . Note that the borrower has a commitment, contingent on not filing for bankruptcy, to repay  $\ell'$  next period.

The household can also save, in which case it commands an interest rate  $\hat{q}_{z'}$ . Note that the exogenous aggregate state that affects the interest rates on savings does so only in a predetermined way. In other words, the interest rate commanded is known at the time of the savings.

Households can save or borrow any amount that they want, but there are endogenously determined upper and lower bounds to their asset holdings. We assume this for now and discuss its verification later. The set of possible asset holdings for the household is  $L = L^- \cup L^+$ .  $L^+$  contains a finite number of positive values, where 0 is the smallest element and  $\ell_{\max}$  is the nonbinding upper bound. The smallest element of  $L$  is  $\ell_{\min} < 0$ . We also denote the entire set of prices by  $q \in \mathbb{R}_+^N$ , where  $N = N_Z \times N_{\tilde{S}} \times N_L$ , and where  $N_Z$ ,  $N_{\tilde{S}}$ , and  $N_L$  refer respectively to the cardinality of the sets  $Z$ ,  $\tilde{S} (= \mathcal{E} \times \Theta \times \Lambda)$ , and  $L$ . In this compact notation we include both the interest rates for borrowers (which depend on  $\{z', \tilde{s}, \ell'\}$ ) and for lenders (notice that interest rates for the lenders depend only on  $z'$ ). Prices are bounded below by 0 (nobody will acquire a liability in exchange for nothing) and above by  $\hat{q}_{z'}$ , the state-dependent risk-free rates of return. We denote the set of possible prices by  $Q = \{q \in \mathbb{R}^N : 0 \leq q_{z',\tilde{s},\ell'} \leq \hat{q}_{z'}\}$ .

### 7.2.3 Households

The preferences of a household are given by the expected value of a discounted sum of instantaneous utility functions,

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, \theta_t) \right\}, \quad (7.1)$$

where  $0 < \beta < 1$  is the discount factor,  $u : \mathbb{R}_+ \rightarrow \mathbb{R}$  is a continuous, strictly increasing, and strictly concave function,  $c_t$  is consumption in period  $t$ , and  $\theta_t$  is the realization of an idiosyncratic shock that affects the marginal utilities of the current period. To be consistent with our description of the timing of events in Fig. 7.1,  $\theta_t$  corresponds to  $\theta'$ .

We look at the household decision in two stages. In the first stage the household decides whether to default (if such a choice is an option) and in the second stage the household chooses how much to borrow or save. In the second stage, the household is in one of four situations: (i) having a good credit rating ( $h = 0$ ) and not having defaulted in this period ( $d = 0$ ), (ii) having a good credit rating ( $h = 0$ ) and having defaulted in this period ( $d = 1$ ), (iii) having a bad credit rating ( $h = 1$ ) and not recovering its credit rating ( $d = 1$ ), and (iv) having a bad credit rating ( $h = 1$ ) but recovering its credit rating ( $d = 0$ ).<sup>5</sup>

The household's characteristics at the time of the savings decision are  $\{\tilde{s}, e, \ell, h, d\}$ , which are the exogenous shocks and earnings, the asset position, the credit history, and the change of the credit status in the current period. The budget set of the household is affected by these characteristics plus the aggregate exogenous state and the loan prices, and we denote it by  $B(z', \tilde{s}, e, \ell, h, d, q)$ . The budget set takes the following form:

1. If the household has a good credit rating ( $h = 0$ ) and has chosen not to default ( $d = 0$ ), then

$$B(z', \tilde{s}, e, \ell, 0, 0, q) = \{c \in \mathbb{R}_+, \ell' \in L : c + q_{z', \tilde{s}, \ell'} \ell' \leq e + \ell - \lambda\}. \quad (7.2)$$

This is the standard case where the household chooses how much to consume and how much to save given that its resources are its inherited assets and its current earnings. Note that on occasion the budget set may be empty, which requires that a combination of bad things have happened to some extent (the household was deep in debt, earnings were low, new loans are expensive, there is large asset destruction).

2. If the household had debt (hence a good credit history) and did choose to default ( $d = 1$ ), then

$$B(z', \tilde{s}, e, \ell, 0, 1, q) = \{c \in \mathbb{R}_+, \ell' = 0 : c \leq (1 - \gamma)e\}. \quad (7.3)$$

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<sup>5</sup>Note that  $d$  is a choice variable for households with a good credit rating ( $h = 0$ ) but is a stochastic variable for households with a bad credit rating ( $h = 1$ ).



In this case, inherited debts (including assets destroyed) did disappear from the budget constraint, saving is not possible, and transaction cost associated with a bad credit history is incurred.

3. If the household had a bad credit rating ( $h = 1$ ) (it did not have an option to default) and its credit rating did not improve ( $d = 1$ ), then

$$B(z', \tilde{s}, \ell, 1, 1, e, q) = \{c \in \mathbb{R}_+, \ell' \geq 0 : c + q_{z', \tilde{s}, \ell'} \ell' \leq (1 - \gamma)e + \max\{\ell - \lambda, 0\}\}. \quad (7.4)$$

With a sustained bad credit rating, the household cannot borrow and is subject to transaction costs and to some assets destruction.<sup>6</sup>

4. If the household had a bad credit rating ( $h = 1$ ) (it did not have an option to default) and its credit rating improved ( $d = 0$ ) then the budget constraint is the same as in case 1,

$$B(z', \tilde{s}, e, \ell, 1, 0, q) = \{c \in \mathbb{R}_+, \ell' \in L : c + q_{z', \tilde{s}, \ell'} \ell' \leq e + \ell - \lambda\}. \quad (7.5)$$

Before we analyze the decision process, it is convenient to describe the situation of the household on the eve of a period, after consumption and saving (or borrowing) have taken place but before the new shocks are realized (those that determine earnings level and asset destruction). The individual state is  $\{z', \tilde{s}, \ell', h'\}$ , where  $h' = d$ . We now define the value of this state for a household by means of the function  $w(z', \tilde{s}, \ell', h'; q) : Q \rightarrow \mathbb{R}$ , which assigns a utility value, given prices  $q$ . We obtain function  $w$  as the unknown in a Bellman-type functional equation that we describe, avoiding most technicalities.<sup>7</sup>

We start by defining an operator that yields the maximum lifetime utility achievable when the household's current earnings draw is  $e$  and its future lifetime utility is assessed according to a given function  $w(z', \tilde{s}, \ell', h'; q)$ . Define  $T_1(w)(z, s, \ell, h; q)$  as follows:

1. For  $(\ell - \lambda) < 0$ ,  $h = 0$ , and  $B(z', \tilde{s}, e, \ell, 0, 0, q) = \emptyset$ , for some  $\{z', \tilde{s}\}$  consistent with

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<sup>6</sup>Note that we are assuming that asset destruction cannot result in households with bad credit histories getting indebted. This assumption excludes the situation where a household that is unable to default is borrowing.

<sup>7</sup>For details see Chatterjee et al. (2002).

$\{z, s\}$ :

$$T_1(w)(z, s, \ell, 0, q) = \sum_{z', \tilde{s}} \tilde{\Gamma}_{z, s, z', \tilde{s}} \{u(e, \tilde{s}) + \beta w(z', \tilde{s}, 0, 1; q)\}. \quad (7.6)$$

2. For  $(\ell - \lambda) < 0$ ,  $h = 0$ , and  $B(z', \tilde{s}, \ell, 0, 0, q) \neq \emptyset$ , for all  $\{z', \tilde{s}\}$  consistent with  $\{z, s\}$ :

$$T_1(w)(z, s, \ell, 0, q) = \max \left\{ \sum_{z', \tilde{s}} \tilde{\Gamma}_{z, s, z', \tilde{s}} \{u(e, \tilde{s}) + \beta w(z', \tilde{s}, 0, 1; q)\}, \right. \\ \left. \sum_{z', \tilde{s}} \tilde{\Gamma}_{z, s, z', \tilde{s}} \left\{ \max_{c, \ell' \in B(z', \tilde{s}, e, \ell, 0, 0, q)} u(c, \tilde{s}) + \beta w(z', \tilde{s}, \ell', 0; q) \right\} \right\}. \quad (7.7)$$

3. For  $(\ell - \lambda) \geq 0$ ,  $h = 0$ :

$$T_1(w)(z, s, \ell, 0, q) = \sum_{z', \tilde{s}} \tilde{\Gamma}_{z, s, z', \tilde{s}} \left\{ \max_{c, \ell' \in B(z', \tilde{s}, e, \ell, 0, 0, q)} u(c, \tilde{s}) + \beta w(z', \tilde{s}, \ell', 0; q) \right\}. \quad (7.8)$$

4. For  $(\ell - \lambda) \geq 0$ ,  $h = 1$ :

$$T_1(w)(z, s, \ell, 1, q) = \eta \left\{ \sum_{z', \tilde{s}} \tilde{\Gamma}_{z, s, z', \tilde{s}} \left\{ \max_{c, \ell' \in B(z, \tilde{s}, e, \ell, 1, 1, q)} u(c, \tilde{s}) + \beta w(z', \tilde{s}, \ell', 1; q) \right\} \right\} \\ + (1 - \eta) \left\{ \sum_{z', \tilde{s}} \tilde{\Gamma}_{z, s, z, \tilde{s}} \left\{ \max_{c, \ell' \in B(z, \tilde{s}, e, \ell, 1, 0, q)} u(c, \tilde{s}) + \beta w(z', \tilde{s}, \ell', 0; q) \right\} \right\}. \quad (7.9)$$

The first part of this definition says that if the household has debt and the budget set conditional on not defaulting may be empty, then the household must default. In this case, the expected lifetime utility of the household is simply the sum of the utility from consuming the current endowment and the discounted expected utility of starting the next period with no assets and a bad credit rating. The second part says that if the household has debt and the budget set conditional on not defaulting is not empty, the household chooses whichever default option yields higher lifetime utility. In the case where both options yield the same utility the household may choose either. The distinction between default under part 1 and default under part 2 is the distinction between “involuntary” and “voluntary” default. In the first case, default is the *only* option, whereas in the second case, it’s the *best* option. The last two parts apply when the household has no debt (so default is not an option), but distinguish between a good and bad credit rating. Note that when the household’s credit rating is bad,

there is some probability  $\eta$  that it will continue in that state in the following period. This way of writing the problem is convenient also from the point of view of describing what is known at the time of each decision.

Note that the image of  $w$  under  $T_1$  has current earnings as an argument, so it is not the same class of function as  $w$ . We denote such an image by  $v(z, s, \ell, h, q; w)$ . To obtain an updated function  $w$  from function  $v$  we have to integrate both over earnings with respect to the c.d.f.  $F(e', \varepsilon', z')$  and with respect to  $\lambda'$ , using transition function  $\pi_{\lambda'|z, \lambda, z'}$ . Chatterjee et al. (2002) shows that an operator constructed this way is well defined and that it is a contraction. This finding allows us to solve the household problem given prices by successive approximations.

#### 7.2.4 Unsecured Credit Industry and Equilibrium

The competitive firms serving the consumer credit industry have access to a credit market in which they can borrow or lend as much as is needed at a risk-free rate  $\hat{q}_{z'}$ . In this environment, firms in the consumer credit industry lend to households at the rate that will yield zero profit, and this is no other than the risk-free rate times the probability that the household does not default. Let  $p_{z', \tilde{s}, \ell'}(q)$  be the probability of default for a household with current shock  $\tilde{s}$  in aggregate state  $z'$  that borrows (or lends)  $\ell'$ , when the set of prices of loans is  $q = \{q_{z', \tilde{s}, \ell'}\}$ . Then competitive equilibrium is defined as follows:

**Definition 1.** *A competitive equilibrium is a set of prices  $q^* = \{q_{z', \tilde{s}, \ell'}^*\}$  such that when agents optimize, taking  $q^*$  as given, the probability of default,  $p_{z', \tilde{s}, \ell'}(q^*)$  satisfies*

$$q_{z', \tilde{s}, \ell'}^* = [1 - p_{z', \tilde{s}, \ell'}(q^*)] \hat{q}_{z'}, \quad \forall z', \tilde{s}, \ell'. \quad (7.10)$$

Chatterjee et al. (2002) show that equilibria for economies such as these exist. A version of their characterization of equilibria for this economy indicates that in any competitive equilibrium (i)  $q_{z', \tilde{s}, \ell'}^* = \hat{q}_{z'}$  for  $\ell' \geq 0$ ; (ii) if the grid for  $L$  is sufficiently fine, for all  $z'$ , there exist  $\tilde{s}$  and  $\ell^0 < 0$  such that  $q_{z', \tilde{s}, \ell^0}^* = \hat{q}_{z'}$ ; (iii)  $q_{z', \tilde{s}, \ell^1}^* \geq q_{z', \tilde{s}, \ell^2}^*$  for  $0 > \ell^1 > \ell^2$ ; and (iv)  $q_{z', \tilde{s}, \ell_{\min}}^* = 0$ .<sup>8</sup> The first property says that the interest rate applied to saving is the risk-free rate, because the consumer credit industry is competitive. The second property says that

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<sup>8</sup>While stating the properties of equilibria we abstract from assets destruction. To account for it is trivial but tedious, except for the case where the mass of agents hit by these shocks varies.

if the grid is taken to be fine enough, there is always a level of debt for which it is never optimal for households to default. As a result, competition leads firms to charge the risk-free rate on these loans as well. The third property says that the price on loans falls with the size of the loans; i.e., the implied interest rate on loans rises with the size of the loan. The fourth property says that the prices of loans eventually become zero; in particular, the price of a loan of size  $\ell_{\min}$  is always zero in every equilibrium, if we set  $\ell_{\min}$  sufficiently small.

Now we are ready to discuss the assumption that there exist endogenous lower and upper bounds of asset holdings. The existence of the lower bound is guaranteed by the third and fourth properties above. Because households will never borrow up to the amount where the price of loans is zero (because households do not gain anything by borrowing up to that amount), the debt level where the price of loans is zero works as a lower bound of asset holdings. The upper bound in assets has a different origin. In models with precautionary savings (and our model is one of them), the existence of an upper bound of asset holdings is implied by the fact that the interest rate is too low in all possible states relative to the rate of time preference. As households increase their wealth, eventually the role of earnings in their income becomes arbitrarily small, whereas this is not the case for the sacrifice that agents have to make to keep saving, and this guarantees the existence of such an upper bound of assets. See Huggett (1993) or Aiyagari (1994) for details.

As we assumed above, prices do not depend on either time or on the distribution of agents: no aggregate uncertainty is revealed in between the instant that loans are issued and the instant in which default is chosen. This means that probabilities of default are not random variables, and that expected profits of firms (zero) coincide with realized profits. Note also that the rates of return of the storage technology are determined exogenously and that are not affected by the distribution of agents, and that probabilities of default only depend on individual variables. This feature also guarantees that prices do not depend on time.

### 7.3 Calibration

We now turn to mapping the model into U.S. data. To do so properly we have to extend the model in two dimensions: the inclusion of depreciation (which only affects accounting) and the explicit consideration of demographics. The latter is more involved and makes it

necessary to generate a large number of agents with low assets.<sup>9</sup> We describe those extensions in Section 7.3.1. Section 7.3.2 maps the statistics of a version of the model without aggregate shocks to the U.S. economy and discusses the extent to which the model achieves those targets. Section 7.4 describes the set of economies with aggregate shocks that we explore.

The actual process by which we map the model to data consists in specifying a set of functional forms and parameter values for which we solve the problem of the agents obtaining decision rules that depend only on exogenous shocks and individual state variables. Then we construct a Markov process with the decision rules and the process for the shocks. We then simulate a large number of individual households histories (up to 1,000,000 to avoid sampling error), which we aggregate to get the whole economy from which we compute statistics. We then compare these statistics to those of the U.S. economy (see Ríos-Rull (1998) for details).

### **7.3.1 Model Extensions for Calibrating Purposes**

We extend the model in two ways that are mostly cosmetic. The first extension is to introduce the depreciation of capital stock. This addition just changes the accounting for the output of the model. It helps to have the appropriate share of consumption and investment out of production.

The second extension is to include population turnover. In this version, households face a constant probability of dying, which generates a constant outflow of households of each type. This is matched by an equal measure of households born every period with zero assets and a good credit history. The initial values of the initial idiosyncratic shocks of the newborns are the same as those of the respective stationary probability distributions. The key difference introduced by this feature of the model is that there are now many agents with a low level of wealth, whereas in a version of the model without population turnover, agents for the most part have large amounts of assets for self-insurance purposes. A large number of households with low wealth implies that some of them will be willing to borrow and then default despite the costly events triggered by these actions.

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<sup>9</sup>Again, see details in Chatterjee et al. (2002).

### 7.3.2 Calibrating the Deterministic Version of the Model to U.S. Data

We now turn to the map between the model and the data. We start describing our targets and their values in the data, and then we describe the extent to which the model matches them and how it does it.

#### The Targets from the Data

The set of statistics that we choose as targets is depicted in the first column of Table 7.1. The second column depicts the long-run averages of these statistics in the U.S. economy. We divide the statistics into four groups. The first group contains the standard aggregate statistics such as wealth-to-output ratio, consumption and investment shares of output, and labor and capital shares of incomes. Those statistics are based on the standard interpretations of NIPA. The risk-free rate of return is chosen to match the average interest rate applied to the checking accounts of commercial banks, because it is the risk-free interest rate for the majority of people who might file bankruptcy.

The second group contains distributional statistics of earnings. We pick four targets related to earnings because we want the earnings process to produce an earnings distribution which is similar to that of the United States. In order to achieve these criteria, we employ an earnings process with five parameters.

The third group of statistics contains the distributional targets, except for earnings. It includes the demographic turnover rate and statistics on the distribution of income and wealth. The demographic turnover rate is set to 2.5%. This implies that the average length of adult life is 40 years, as a compromise for an economy without population growth. We use Gini indices of income and wealth as statistics representing the distribution.

The fourth group of statistics are default-related. According to the Administrative Office of the U.S. Courts, the proportion of the households that filed consumer bankruptcy to the total number of households is around 1.3% in recent years. Among them, Chapter 7 bankruptcy, which the model is intended to capture, makes up more than 70%.<sup>10</sup> Therefore we decided to use 1.0% as our target for the proportion of households declaring bankruptcy.

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<sup>10</sup>For a more detailed description of U.S. bankruptcy law and data on bankruptcies, see Chatterjee et al. (2002).

Table 7.1: **Statistics of the Deterministic Economy.**

Statistic	Target	Model
<b>Basic aggregate targets</b>		
Wealth-to-output ratio	3.32	3.35
Labor share	0.64	0.64
Risk-free rate of return	0.5%	0.5%
<b>Earnings distribution related targets</b>		
Earnings Gini index	0.61	0.61
Bottom 40% earnings	3.8%	7.6%
Fourth quintile earnings	22.9 %	19.4%
Top quintile earnings	60.2 %	63.4%
<b>Other distributional targets</b>		
Population turnover rate	2.5%	2.5%
Income Gini index	0.55	0.56
Wealth Gini index	0.80	0.80
<b>Default related targets</b>		
Households filing bankruptcy	1.0%	1.0%
Average length of punishment	10 years	10 years
Households with negative assets	9.9%	10.0%

Source: The source for the distributional data is Budría et al. (2002).

The average length of punishment is chosen to be 10 years because the U.S. Bankruptcy Code states this as the length of time for which having filed for bankruptcy stays in the credit rating. In the model we report the average length of bad-credit-rating spells.

### The Targets in the Model

To get the model economy to achieve these targets, we essentially follow the procedures described in Chatterjee et al. (2002), adjusted for the fact that this model economy does not exclude any subset of the population. There are four types of individual shocks to be specified, namely the shock governing the earnings process ( $\varepsilon$ ), the earnings shock ( $e$ ), the preference shock ( $\theta$ ), and the asset destruction shock ( $\lambda$ ). We choose earnings to be independently and identically distributed (i.i.d.) (meaning that the shock  $\varepsilon$  is irrelevant) with the shape given by Fig. 7.2. This is a four-parameter distribution. In addition there are two more parameters, the upper and lower bounds for earnings. Of these two, one should not matter, as the model is unit independent. This gives a total of five parameters for the distribution of earnings.

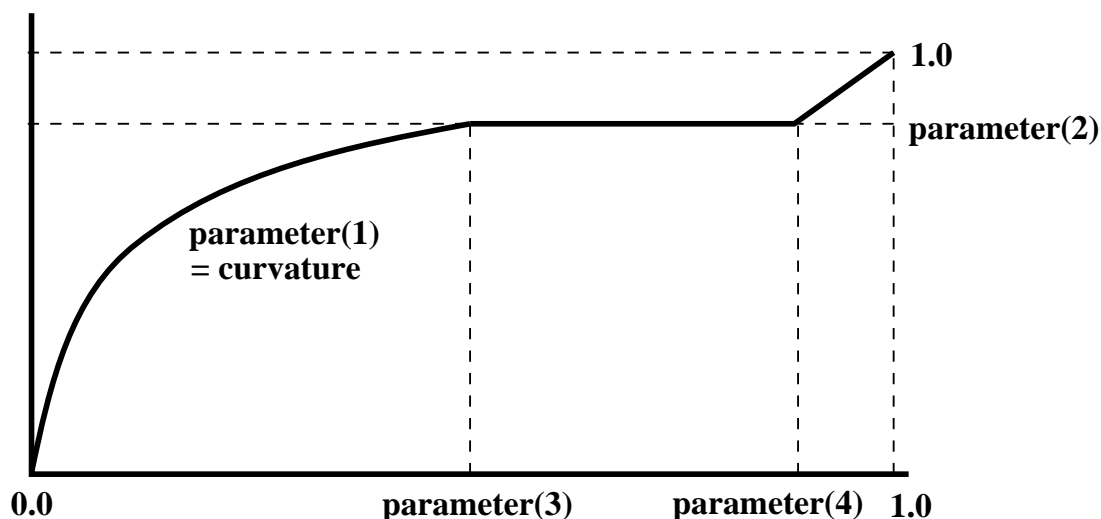


Figure 7.2: Nature of the earnings process.

As for the preference shock, we assume that it is i.i.d., so it is characterized by two parameters (magnitude and probability of the bad shock). To calibrate our baseline model we



abstract from the asset-destruction shock. We will evaluate the effect of the asset-destruction shock later.

In total, there are 14 parameters that have to be pinned down. They are (i) five parameters characterizing the earnings process, (ii) two parameters characterizing the preference-shock process (magnitude and probability of the bad shock), (iii) two preference parameters (time-discount factor and degree of risk aversion<sup>11</sup>), (iv) two parameters specifying bankruptcy law (average length of punishment and cost of having a bad credit history), (v) one demographic transition parameter (population turnover rate), and (vi) two parameters specifying technology (risk-free interest rate and depreciation rate). As there are 13 target statistics in Table 7.1, we set the degree of risk aversion and fix its value to 2.0 to have 13 parameters, and we check the robustness of our results to a change in the value of the risk-aversion parameter later.

The last column of Table 7.1 shows the values of target statistics in the steady state version (i.e., without aggregate shocks) of the model economy. A comparison between the last two columns gives us a sense of the fit of the model to data. It is immediate to see that the statistics of the model economy match their counterparts in the U.S. economy surprisingly well. Especially, the model is successful in replicating the very large number of bankruptcies and the large number of households in debt that are observed in recent years. It is very hard to get simultaneously a high amount of saving (the capital-output ratio is 3.3), a large number of households in debt (around 10% of total number of households), and a large number of defaults simultaneously, even though we add a demographic turnover. As Castañeda et al. (2003) show, simple models with agents differing only in the realizations of uninsurable shocks to earnings have a hard time matching the U.S. wealth distribution. They also show that a large wealth dispersion is achieved by posing very skewed earnings and by modeling households not as purely life-cycle agents but as members of a dynasty. We basically follow their strategy in finding the set of parameters that yields statistics that are closest to the data. It is well worth mentioning that the inequality of earnings, income, and wealth in terms of the Gini index in the model is very similar to the data. Income is less concentrated than earnings, whereas wealth is more concentrated than earnings both in the model and in the data.

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<sup>11</sup>We use the standard constant relative risk aversion (CRRA) period utility function.

## 7.4 Model Economies with Aggregate Shocks

We now turn to exploring the business-cycle behavior of the stochastic model economies. There are various ways of introducing business cycles via aggregate shocks and we explore many of them. We start by looking at an economy where the business-cycle variation affects only the level of earnings, which we denote as the baseline model economy. In the economy, earnings of all households are uniformly higher in an expansion than in a recession. At the same time, we keep the long-run average of aggregate earnings at the same level as in the deterministic economy. We do so by multiplying the earnings of all the households by a number which is a function of the aggregate state of the economy. We use two aggregate states, which is the simplest way to represent expansions and recessions. This implies that we need to pin down three parameters: one for the amplitude of the aggregate shock and two for its persistence. We set the following three targets to calibrate this economy: (i) the volatility of this economy is like that of the U.S. economy, measured in terms of the standard deviations of the logs of HP residuals with parameter 100, (ii) the average duration of recessions is 2 years, and (iii) the average duration of expansions is 10 years. This is in a sense a model of the business cycle that completely ignores one of its important properties: that not all households fare the same in recessions and in expansions. Section 7.5 reports and discusses the properties of the baseline model economy. We then explore the business-cycle behavior of other model economies that have different mechanisms to generate fluctuations in economic activity. Especially, our emphasis is on to what extent these model economies can fix the defects of the baseline economy.

The other economies that we run in addition to those of (i) the baseline model are (ii) an economy with small interest-rate shocks, (iii) an economy with large interest-rate shocks, (iv) an economy with a small asset-destruction shock, (v) an economy with a large asset-destruction shock, and (vi) an economy with all the above (small) shocks. We also explore the properties of (vii) an economy with symmetric i.i.d. aggregate shocks and (viii) an economy with smaller risk aversion.

The next economy we explore, economy (ii) has interest-rate shocks. We want to know how the economy responds to interest-rate hikes and we want to see if in any way they tend to reduce economic activity (which, here, is savings). This economy is like the baseline economy but with the additional feature of interest-rate fluctuations. Its risk-free interest

rates are 0.75% in expansions and 0.25% in recessions, making interest rates procyclical.<sup>12</sup> We retain the same transition matrix for the aggregate shocks as the baseline. Still, we have to specify the magnitude of the variations in earnings. We adjust it so that the implied volatility of output is the same as in the baseline model economy.

The next economy, economy (iii), is identical to economy (ii) except for the values for interest rates: its interest rates are 1.0% in expansions and 0.0% in recessions. All the other features are the same as in economy (ii), implying that in the economy (ii) earnings are less volatile than in the baseline economy but more volatile than in the economy (iii).

Next, in economy (iv) we add to the baseline an asset-destruction shock to some households, of the magnitude of 100% of mean earnings, with i.i.d. probability of 0.1% in expansions and 1.0% in recessions. Economy (v) pumps up these shocks to be 300% of mean earnings. Again, we retain the transition matrix of the earning-level shock in the baseline model and adjust the magnitude of the shock in order to match the volatility of the output. We do this because we interpret these shocks as business losses, not as natural disasters.

Economy (vi) has both small-interest rate shocks and small asset-destruction shocks as above. Again, we adjust the magnitude of the earnings shock to match the volatility of output.

Economies (vii) and (viii) are used to check the robustness of our results from the baseline model economy. The economy (vii) is used to show the importance of our assumption on the persistence of expansions and recessions for our main results. Outputs from the economy (viii) tell how robust the properties of the baseline model are to a change of the risk-aversion parameter, which we set to 2.0 in our baseline economy.

In the economy (vii), the aggregate shock is set to i.i.d. The three parameters that govern the aggregate shocks are set by letting  $\Gamma_{zz'} = 0.5$  for all  $z$  and all  $z'$ , and we choose the difference between the two values of  $z$  to set the volatility of output equal to that in the data, keeping the mean earnings at the same level as the baseline economy.

Finally, economy (viii) is like the baseline, but its risk-aversion parameter is set to 1.2

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<sup>12</sup>The correlation between HP-filtered output and the 1-month Treasury bill rate is 0.40 according to Cooley and Hansen (1995).

Table 7.2: U.S. Economy: Deviation from Trend, 1948-1986

Variable	SD%	Relative to SD% of output	Cross-correlations of output with				
			$x(-2)$	$x(-1)$	$x$	$x(+1)$	$x(+2)$
Output	2.63	1.00	0.02	0.56	1.00	0.56	0.02
Consumption	1.27	0.48	-0.16	0.39	0.78	0.53	-0.19
Investment	7.86	2.98	0.07	0.48	0.70	-0.01	-0.33
Labor Share	0.66	0.25	-0.42	-0.41	-0.10	0.39	0.30

Source: Castañeda et al. (1998).

(almost log) instead of the value of 2.0 used in all the other experiments. Again we adjust the magnitude of the earnings shocks so that the mean and the volatility of earnings are at the same level as in the baseline economy.

## 7.5 The Business Cycle Behavior of the Basic Model Economy: Aggregate Shocks to the Level of Earnings

### 7.5.1 Business Cycle Statistics

To compare with the properties of the model economies, Table 7.2 shows the properties of the aggregate fluctuations at a yearly frequency in the U.S. economy. These properties are well known and need not be repeated here.<sup>13</sup> We now describe the business-cycle properties of the basic model with the business cycle affecting earnings of all households in the same proportion.

Table 7.3 reports the main statistics regarding the volatility of the baseline model economy. This table shows the value of the statistics and the standard deviations of those statistics (in parentheses) over nine samples. Each sample has a length of 200 periods (after the first 100 periods are dropped).<sup>14</sup> We see that, in this model economy, some of

<sup>13</sup>Both the data and the series from the various model economies have been detrended using the HP filter with parameter 100.

<sup>14</sup>We choose this large length instead of the 40 periods or so of available data because we are exploiting

the standard features of the real business cycle models remain the same, while others are somehow different. Our summary of the findings on basic aggregate statistics of the baseline model economy is as follows:

1. Investment is very volatile, more so than output. Consumption is not very volatile, less so than output. Both are strongly procyclical. These properties are common to most real business cycle models and to the data.
2. The volatility of consumption is substantially smaller than in the data.
3. Both the correlation between output and consumption and the correlation between output and investment are close to one, much higher than in the data. This is not surprising given that output in the model has only these two components and in the data there are public expenditures and net exports.
4. Earnings are quite volatile, as they are the main engine of business cycles.
5. Capital income is surprisingly volatile given that the interest rates of savings are constant. This is entirely due to the high volatility of aggregate asset holdings.
6. Labor share is quite volatile, more so than in the data. It is strongly procyclical, whereas it is slightly countercyclical in data. This is an artifact of the definition of the cycle based on earnings variations while rates of return are constant.

Table 7.3 also reports the business-cycle properties of the bankruptcy-related statistics. Although these statistics do not have a U.S. economy counterpart because of the availability of data, nor are there outputs from other models to compare with, the output can tell us the mechanism of the model and suggest the direction in which the model should be extended. We list the findings below and discuss them in the next section.

1. The volatility of the number of defaulters is quite large. The standard deviation of the log of the proportion of households declaring bankruptcy is almost four times that of the log of output. The corresponding ratio for the proportion of delinquent households is 60%.

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the behavior of the model economies and we do not want a lot of sampling error. This makes the model and the data not strictly comparable.

Table 7.3: Cyclical Behavior of the Baseline Model

Variable	SD%	Relative to SD% of output	Auto- corr	Cross-correlations of output with				
				$x(-2)$	$x(-1)$	$x$	$x(+1)$	$x(+2)$
Output	2.64 (0.33)	1.00	0.24 (0.07)	-0.12 (0.07)	0.24 (0.07)	1.00 (0.00)	0.24 (0.07)	-0.12 (0.07)
Consumption	0.74 (0.08)	0.28	0.64 (0.04)	-0.17 (0.06)	0.47 (0.02)	0.87 (0.01)	0.42 (0.07)	0.08 (0.09)
Investment	6.63 (0.86)	2.51	0.13 (0.07)	-0.10 (0.07)	0.18 (0.07)	0.99 (0.00)	0.19 (0.06)	-0.16 (0.06)
Asset holding	1.39 (0.19)	0.53	0.36 (0.07)	-0.24 (0.09)	-0.44 (0.05)	-0.67 (0.04)	0.33 (0.03)	0.57 (0.03)
Earnings	4.35 (0.54)	1.65	0.21 (0.07)	-0.07 (0.07)	0.28 (0.07)	1.00 (0.00)	0.17 (0.06)	-0.18 (0.06)
Capital income	0.80 (0.10)	0.30	0.64 (0.05)	-0.41 (0.06)	-0.44 (0.04)	-0.26 (0.04)	0.57 (0.01)	0.66 (0.03)
Labor share	1.74 (0.21)	0.66	0.20 (0.07)	0.01 (0.08)	0.33 (0.07)	0.97 (0.01)	0.06 (0.05)	-0.28 (0.04)
Defaulting pop	10.28 (1.30)	3.89	0.27 (0.06)	0.02 (0.09)	0.33 (0.07)	0.93 (0.01)	0.10 (0.03)	-0.41 (0.05)
Delinquent pop	1.60 (0.19)	0.61	0.66 (0.04)	-0.23 (0.05)	0.05 (0.07)	0.75 (0.01)	0.72 (0.03)	0.30 (0.08)
Pop in debt	1.80 (0.20)	0.69	0.59 (0.05)	0.24 (0.09)	0.44 (0.05)	0.67 (0.04)	-0.06 (0.03)	-0.52 (0.01)
Debt stock	22.38 (2.88)	8.47	0.18 (0.06)	-0.01 (0.08)	0.31 (0.07)	0.96 (0.01)	0.06 (0.05)	-0.31 (0.05)

2. The number of borrowers is two-thirds as volatile as the output.
3. The stock of debt is extremely volatile, more than twice as volatile as the proportion of defaulters, which is the next most volatile variable.
4. The number of bankruptcy filings is procyclical. The aggregate amount of debt, as well as the number of households in debt, is also procyclical.

### 7.5.2 Discussion of the Bankruptcy-Related Statistics

The volatilities of the bankruptcy-related statistics are high, which indicates that they are likely to be an important element characterizing business cycles. Unfortunately bankruptcy filings are *procyclical*, a highly counterfactual feature, and indeed a counterintuitive one, at least at first glance. However, note that an expansion may be a good time to default, because the aggregate state is persistent, and hence the future looks better and the likelihood of needing the option to borrow is small. In addition, loans are very volatile and strongly positively correlated with output.<sup>15</sup> So we have that expansions bring not only good economic conditions, but also more debt and a good outlook, a recipe for a hike in the number of defaulters. We have to ask, then, why a forthcoming expansion induces an increase in loans, at a time when they are perhaps less needed. The reason is that the price of loans goes down dramatically. Although the average borrowing interest rate weighted by the number of borrowers is 22% in expansions and 16% in recessions, the interest rates applied to the same types of loans are moving in opposite directions. For example, the interest rate applied to a loan whose amount is about one-eighth of average income is 7% in expansions and 17% in recessions. Fluctuations of interest rates applied to various sizes of loans are displayed in Fig. 7.3. It shows the interest rate schedule of loans offered to the same types of households in expansions and recessions, and in the deterministic economy.

Given that expansions and recessions are perfectly forecast the probabilities of default are perfectly known and interest rates adjust accordingly. For example, interest rates for loans go down in expansions, as the decrease in the number of defaults in the next period is forecast by the loan industry. In equilibrium, then, an expansion induces an enormous

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<sup>15</sup>The correlation between initial debt and output is 0.96. Note that loans were taken the period before, but the current period's state of the economy was known at the time.

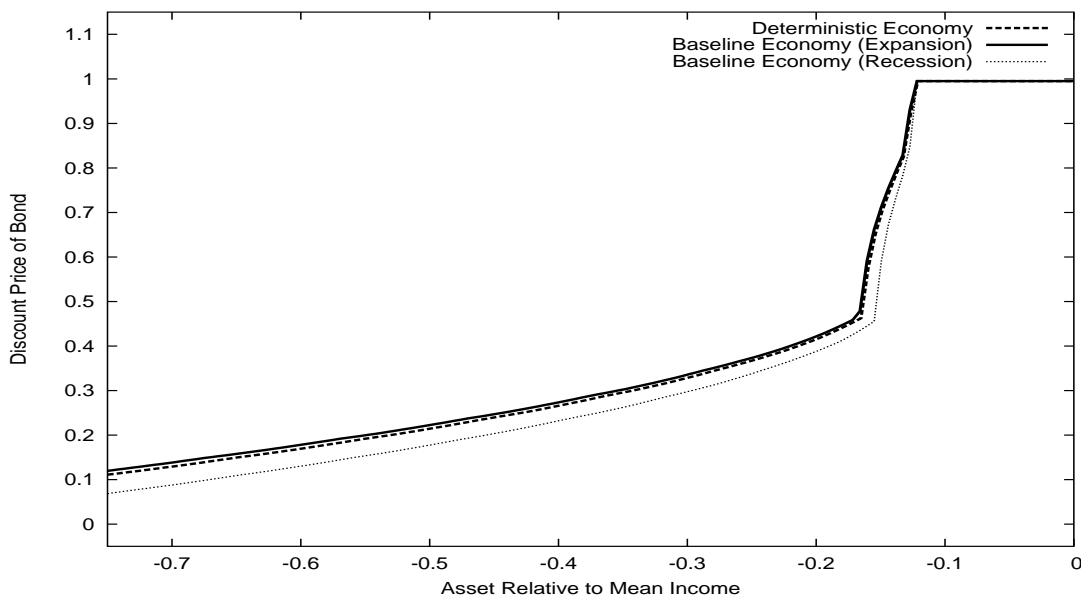


Figure 7.3: **Price of loans for the baseline economy.**

increase in debt and a smaller increase in default. In fact, defaults at each loan size go down, allowing interest-rate reductions that are responsible for the increase in debt.

We conjecture that this type of behavior is an artifact of the timing of events in the model. Perfectly forecastable changes in the aggregate state translate into huge movements in interest rates and in debt volumes that override any negative effect of recessions on bankruptcy filings. We think that there are two possible mechanisms to change this counterfactual behavior of bankruptcy filings. One of these mechanisms is the introduction of aggregate shocks that surprise agents, making interest rates much more smooth and hence debt much less volatile. This is part of our ongoing research agenda. The other mechanism that might work is to consider a type of shock that pushes households into debt and that is larger or more frequent in recessions. In this sense, we presume that domino effects in the economy might play an important role in producing the countercyclical nature of the number of bankruptcies. For simplicity, we take shocks to the asset position as a proxy for domino effects in this paper and leave a more careful treatment of domino effects for future research.



## **7.6 The Business-Cycle Behavior of the Other Model Economies**

In this section we explore the business-cycle properties of the other model economies that were described in Section 7.4. Table 7.4 summarizes the business-cycle statistics of the standard aggregates. Table 7.5 shows the business-cycle statistics related to default and debt. We proceed by comparing these model economies both with the data and with the baseline model economy. Recall that all those experiments are designed to have the same output volatility.

### **7.6.1 The Business-Cycle Behavior of Economies with Interest-Rate Fluctuations**

The third and fourth rows of Table 7.4 show that the business-cycle properties of models with interest-rate shocks are quite similar to those of the baseline model. Interest-rate movements increase the volatility of investment and reduce that of consumption. Capital income becomes more volatile, which reduces the volatility of the labor share to a level that is close to the data, and makes labor share countercyclical, as in the data. This is simply because capital income is now strongly procyclical due to the procyclical nature of interest-rate movement.

As for the cyclical behavior of bankruptcy-related statistics, Table 7.5 shows that the effect of additional interest-rate shocks relative to the baseline is to scale down the volatility and the correlations of all the statistics, while keeping all the qualitative characteristics the same. This result is expected because the exogenous procyclical interest-rate shocks reduce the movements of the interest rates faced by borrowers, counteracting the countercyclicity of the interest rates faced by borrowers in the baseline.

### **7.6.2 The Business-Cycle Behavior of Economies with Asset-Destruction Shock**

According to the fifth and sixth rows of Table 7.4, adding asset-destruction shock does not induce any significant changes in the business-cycle statistics of the main macroeconomic aggregates relative to the baseline model economy. As in the models with interest-rate shocks, the correlation between consumption and output and the volatility of investment become a little closer to the data. The cyclical properties of labor share are virtually the same as in the baseline economy.

On the other hand, shocks to asset level drastically change the business-cycle properties of bankruptcy-related statistics. It is immediate to see from Table 7.5 that the correlations between output and the numbers of defaulters, delinquents, and households in debt are lowered by the introduction of asset-destruction shock. They are strongly countercyclical in the case of the economy with large asset-destruction shocks. The property of the model that households face higher interest rates and thus can borrow less in recessions remains, as we can see from the fact that the aggregate level of debt is still procyclical in models with asset-destruction shocks. But because the shock hits more households in debt and they are more likely to default in recessions, the numbers of households that are indebted, declaring bankruptcy, and delinquent increase in recessions, making them countercyclical.

However, the volatility of debt is still quite high and procyclical, the reason again being the perfect predictability of the default rates. The bottom line is that although the models with asset-destruction shocks show more intuitive relations between output and the number of defaulters, they do so only with very high aggregate shocks and simultaneously with large procyclicality of debt.

### **7.6.3 The Business-Cycle Behavior of Other Economies**

The model with both small interest-rate shocks and small asset-destruction shocks shows a result that is a mixture of the two models with one of the two shocks (see the seventh row of Table 7.4 and the sixth row of Table 7.5), which implies that some (the correlation between consumption and output, the volatility of labor share) move a little closer to the data, while others (the volatility of investment, the volatility of consumption) do not change much compared with the baseline model. The correlation between the number of defaulters and the output is almost zero, due to the effect of the asset-destruction shocks, but all four default-related statistics are procyclical, as in the baseline.

The properties of the models that we study in this paper are robust to the changes in two of our assumptions, that the risk aversion parameter is 2.0 and that the average durations of expansions and recessions are 10 and 2 years, respectively. Both in a model with symmetric i.i.d. aggregate shocks and in a model with lower risk aversion the volatility of debt is very high compared to that of other bankruptcy-related variables, which are strongly procyclical, as they are in the baseline economy. These are shown in the last two rows of Tables 7.4

and 7.5. The only noticeable difference is that in the economy with lower risk aversion (and hence higher intertemporal elasticity of substitution), both debt and filings move less than in the baseline.

## 7.7 Concluding Remarks

In this paper we have extended the work of Chatterjee et al. (2002) to include aggregate real shocks to economic activity in economies with storage technology. We have shown that the results from the economies studied here lack some interesting aspects regarding bankruptcy. This is because there is no uncertainty in the number of bankruptcies, and hence interest rates on loans completely bear the cyclical changes. This feature also prevents the existence of domino effects. The sources of the shortcomings of the present paper are technical: we have manipulated the timing in the model to prevent the distribution of agents from being important in forecasting economic conditions relevant for the agents (see Ríos-Rull (1998), Krusell and Smith (1997), or Krusell and Smith (1998)) in a way similar to that of Diaz-Gimenez et al. (1992).

In this sense, we see this paper as a progress report toward exploring economies that can be more seriously compared to the U.S. economy. The economy must have a production sector. Aggregate uncertainty should be somewhat unpredictable to generate surprising changes in the volume of default. We are currently working in using modern techniques to overcome these shortcomings and address economies with a more relevant production structure and with the possibility of a domino effect, in which the default of some agents is what triggers the default of others.

We have shown in this paper how the model economies account for the very high number of bankruptcies in recent years, while at the same time they replicate many other main statistics of the macroeconomic aggregates of the U.S. economy. We have analyzed the business-cycle properties of a variety of calibrated models that differ in how we generate business cycles. We have found that the numbers of defaulters, delinquents, and borrowers are quite volatile relative to output fluctuations. We have also found that the existence of negative shocks to wealth during recessions is an important mechanism in generating a more intuitive (i.e., negative) correlation between number of bankruptcies and output. We are excited about the prospect of our research telling us how important this mechanism may be

Table 7.4: Experiments Result 1: Aggregate Fluctuations

Economy	SD relative to SD of output						Correlation with output					
	Cons	Inv	Asset	Earn	$rK$	LSh	Cons	Inv	Asset	Earn	$rK$	LSh
U.S.	0.48	2.98	-	-	-	0.25	0.78	0.70	-	-	-	-0.10
Baseline	0.28	2.51	0.53	1.65	0.30	0.66	0.87	0.99	-0.67	1.00	-0.26	0.97
Interest-rate shock ( $\pm 0.25\%$ )	0.22	2.66	0.44	1.27	0.61	0.30	0.79	1.00	-0.49	0.99	0.92	0.88
Interest-rate shock ( $\pm 0.5\%$ )	0.24	2.61	0.37	0.89	1.21	0.16	0.81	0.99	-0.33	0.99	0.99	-0.74
Asset-destruction shock (100%)	0.25	2.56	0.51	1.67	0.32	0.69	0.74	0.99	-0.58	0.99	-0.22	0.96
Asset-destruction shock (300%)	0.21	2.76	0.46	1.62	0.35	0.64	0.69	0.99	-0.46	0.99	-0.17	0.95
All shocks	0.25	2.49	0.42	1.27	0.62	0.30	0.83	0.99	-0.45	0.99	0.93	0.88
I.i.d. aggregate shock	0.23	2.50	0.53	1.64	0.24	0.65	0.90	1.00	-0.77	1.00	-0.44	0.98
Lower risk aversion ( $\sigma = 1.2$ )	0.27	3.01	0.70	1.50	0.35	0.52	0.84	0.99	-0.72	1.00	-0.26	0.96

Note:  $rK$  and  $LSh$  denote capital income and labor share of output, respectively.

Table 7.5: Experiments Result 2: Business-Cycle Statistics of Bankruptcy Behavior

Economy	SD relative to SD of output				Correlation with output			
	Default	Delinq	Pop debt	Debt	Default	Delinq	Pop debt	debt
Baseline	3.89	0.61	0.69	8.47	0.93	0.75	0.67	0.96
Interest-rate shock ( $\pm 0.25\%$ )	2.63	0.41	0.58	5.99	0.90	0.79	0.63	0.92
Interest-rate shock ( $\pm 0.5\%$ )	2.52	0.37	0.41	5.40	0.87	0.84	0.41	0.89
Asset-destruction shock (100%)	1.05	0.25	0.66	6.80	0.24	0.48	0.20	0.93
Asset-destruction shock (300%)	3.84	0.67	0.90	3.94	-0.97	-0.57	-0.38	0.93
All shocks	0.93	0.22	0.58	5.71	0.01	0.39	0.03	0.92
I.i.d. aggregate shock	3.54	0.40	0.41	8.52	0.92	0.86	0.52	0.91
Lower risk aversion ( $\sigma = 1.2$ )	1.76	0.32	0.72	3.67	0.86	0.65	0.76	0.96

Note: Default, Delinq, and Pop debt in the table denote the proportion of households that declare bankruptcy, the proportion of delinquent households, and the proportion of households with negative asset level, respectively. Debt denotes aggregate debt level of the economy.

when we look at economies with production and with uncertainty in the profits of lenders as well as other economic activities.

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