Worker Flows and Job Flows: A Quantitative Investigation*

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Abstract

This paper studies quantitative properties of a multiple-worker firm search/matching model and investigates how worker transition rates and job flow rates are interrelated. We show that allowing for job-to-job transitions in the model is essential to simultaneously account for the cyclical features of worker transition rates and job flow rates. Important to this result are the distinctions between the job creation rate and the hiring rate and between the job destruction rate and the layoff rate. In the model without job-to-job transitions, these distinctions essentially disappear, thus making it impossible to simultaneously replicate the cyclical features of both labor market flows.

Keywords: job flows, worker flows, multiple-worker firm, and search and matching
JEL Classification: E24, E32, J63, J64

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1 Introduction

The flow analysis is now a standard tool to analyze the labor market dynamics. Measures of job creation and destruction rates, proposed by Davis et al. (1996), are constructed from establishment-level employment observations and are supposed to capture rates at which new jobs are created and existing jobs are destroyed. Cyclical features of these variables have been extensively studied and are well known. For example, the job creation rate is procyclical, while the job destruction rate is countercyclical. Worker transition rates, measured from a survey of households, capture similar but distinct labor market information. Again, researchers have studied their cyclical features and find that the job finding rate for unemployed workers is procyclical and the separation rate into unemployment is countercyclical.

The macro-labor literature has used either job flow rates or worker transition rates as a yardstick to evaluate quantitative performance of labor search/matching models. For example, Mortensen and Pissarides (1994) and den Haan et al. (2000) focus on job flow rates in evaluating their models' quantitative performance. Since an influential paper by Shimer (2005), however, the literature has focused mostly on the model's performance in replicating the cyclical features of worker transition rates.

In this paper, we consider both job flow rates and worker transition rates simultaneously within one framework. As mentioned above, the overall empirical regularities of these data are well established and tell us intuitive stories about what is happening in the labor market. For example, a higher job destruction rate is likely to imply the deterioration of the labor market. A similar story can be told when the transition rate from employment to unemployment spikes up. As we will show in this paper, however, the relationships between these flow variables are more complex and richer than these casual observations suggest. Therefore, there is much to learn from considering both flow measures simultaneously, and the simultaneous analysis allows us to disentangle different underlying forces at work in driving each of these variables, thereby giving us a richer and better understanding of the U.S. labor market dynamics.

For the purpose of this paper, we consider a search/matching model in which each firm operates a decreasing returns-to-scale production technology and is subject to aggregate as well as idiosyncratic productivity shocks. Each firm hires multiple workers, in contrast to the canonical matching model of Mortensen and Pissarides (1994), in which a worker-firm match is taken to be the unit of analysis. Cooper et al. (2007) and Elsby and Michaels (2013) also study the multiple-worker firm environment. Our paper is different from these existing papers because we explicitly incorporate job-to-job transitions into the model. Both the multiple-worker firm environment and job-to-job transitions are essential for analyzing both worker transition rates and job flow rates. To see this point, note that job flows are defined by establishment-level *net* employment changes over a quarterly period: Job creation aggregates net employment changes at the establishments that are expanding on net over the period, and, similarly, job destruction is the sum of net employment losses. To be consistent with the measurement, we need a model with a meaningful notion of the establishment that hires many workers. Furthermore, in the environment in which job-to-job transitions are assumed away, the difference between net employment changes and gross worker flows is largely futile

except for the difference that arises to due to the different data collection frequency.[1] As a simple example, consider a firm that plans to expand its employment in the current period. In the absence of worker turnover due to quits, the number of hires is identical to net jobs created in this period. However, job-to-job transitions introduce *endogenous* worker turnover and thus work as a wedge between gross hires and net employment gains. The firm may lose some of its workers to other firms and thus net employment gains would differ from gross hires. Importantly, the pace of job-to-job transitions in the model is time varying and thus the wedge is also time varying. In the paper, we analyze various cases in which net employment changes and gross flows are different.

We calibrate the model by matching the steady-state levels of worker transition rates and cross-sectional moments such as the dispersion of the employment growth distribution reported by Davis et al. (2010). We show that the calibrated model captures essential features of the cross-sectional "hockey stick" relationships between gross worker flows and net employment growth studied by Davis et al. (2012). We show that the model successfully replicates overall cyclical features of worker transition rates and the job creation and destruction rates. As noted above, incorporating job-to-job transitions into the model plays a crucial role for this result. Suppose that the firm aims to reduce its workforce size. How many workers the firm aims to lay off depends on how many workers quit the firm. Those who are laid off flow into unemployment, while those who quit make job-to-job transitions, implying that the transition rate from employment to unemployment (EU transition rate) is different from the job destruction rate. It is also possible that this firm actually hires some workers when the number of quits exceeds the total number of desired reduction of employment. The relationship between the job creation rate (sum of net employment gains normalized by aggregate employment) and the overall hiring rate (all hires normalized by aggregate employment) is even more complex. As discussed previously, the presence of job-to-job transitions makes net employment change and gross hires different from each other. There is an important aggregate implication of this fact. Suppose (for the sake of an illustration) that one expanding firm hires all new workers from another expanding firm. In this case, the latter expanding firm must be hiring more workers than it lost to "create new jobs." If, on the other hand, those two firms hire all of their new workers from the unemployment pool or shrinking firms, net job gains are equal to total hires. These examples imply that the relationship between net job gains and total hires depends on the pace of job-to-job transitions and the share of job-to-job transitions that occur within expanding firms, both of which are endogenously changing over time along with the aggregate shock.

Moreover, the example in the previous paragraph shows how a chain of hiring occurs when workers make job-to-job transitions from one expanding firm to other expanding firms. That is, as the pace of job-to-job transitions increases, the number of new hires necessary to achieve the target employment size increases. We show that the chain of hiring is reflected in the behavior of vacancies in our model in which vacancies are more procyclical, volatile,

[1] Job flows are measured by net employment changes over a quarterly period, whereas worker transition rates are measured from the monthly survey. We address this issue by solving the model at monthly frequency and analyzing job flows constructed in the same way as in the actual data.

and persistent compared with the behavior in the model without job-to-job transitions. The result in our model captures the notion of "vacancy chain" introduced by Akerlof et al. (1988).

Let us now discuss where our paper stands in relation to the literature. As mentioned above, this paper is closely related to the studies by Cooper et al. (2007) and Elsby and Michaels (2013), who also quantitatively analyze a multiple-worker firm matching model. The key difference from these papers is the presence of job-to-job transitions in our model. There are several other papers that study the directed search environment with decreasing returns to scale (e.g., Kaas and Kircher (2013) and Schaal (2012)). Kaas and Kircher analyze the environment without job-to-job transitions, and Schaal adds on-the-job search to the model. However, Schaal focuses more on the recent Great Recession episode in the presence of the uncertainty shock. On the other hand, our paper follows the random matching tradition more closely and looks more generally at the model's cyclical features.[2]

In terms of economic interest, our paper is related to the work by Mortensen (1994), who also attempts to explain the cyclicality of worker transition rates and job flow rates. However, he explores this topic in a single-worker firm matching model with on-the-job search, and thus faces the limitations we discussed above. Veracierto (2009) provides a synthesis of the different strands of literature (in particular, the Mortensen-Pissarides random-matching framework and the Lucas and Prescott (1974) island framework) and discusses the cyclical properties of worker transition rates and job flow rates in the model. However, his model does not allow for job-to-job transitions. Our analysis of the relationships between worker transition rates and job flow rates is a key part of our paper that is distinct from his paper.[3]

This paper proceeds as follows. In the next section, we summarize the business cycle features of worker transition rates and job flow rates. Section 3 presents our model. Section 4 summarizes the solution algorithms to numerically solve the model. The details are presented in the Appendix. Section 5 discusses the calibration. Section 6 discusses the paper's main results. To demonstrate the importance of job-to-job transitions in our model, we compare the performance of our model with that of the model without job-to-job transitions. The final section concludes by discussing some limitations of our paper.

2 Empirical Facts

This section reviews the cyclical properties of worker transition rates and job flow rates. While one can find the cyclical properties of these series in the literature, the two sets of data are usually discussed in isolation. Let us first review the definitions of the series.

[2] A recent work by Moscarini and Postel-Vinay (2013) analyzes and solves the random-matching, wage-posting model under the presence of the aggregate shock. But worker transitions into unemployment are exogenous, and their research interest is different from ours.

[3] Note, however, that his model is a full-fledged real business cycle model with physical capital and risk aversion. He can therefore assess the broader macroeconomic implications of this model. See also Veracierto (2007), who studies normative aspects of a similar environment without aggregate uncertainty.

2.1 Measurement

Job flow rates. The job flow series are measured from the Business Employment Dynamics (BED) data, which are based on the administrative records of the Quarterly Census of Employment and Wages (QCEW).[4] These measures were originally developed by Davis et al. (1996): Job creation (destruction) is defined as the sum of net employment gains (losses) over all establishments that expand (contract) or start up (shut down) between the two sampling dates. Because we are interested in business cycle fluctuations of the series, we use the series that trace net employment changes over a quarterly period. Normalizing creation and destruction by aggregate employment yields job creation and destruction *rates*, respectively.[5] In this paper, we sometimes use the term "job flows" as a generic term, but the specific empirical measures we aim to explain are always job flow rates. The sample period of the series starts at 1992Q3 and ends at 2011Q4.

Worker transition rates. We use the Current Population Survey (CPS) to measure worker transition rates. The CPS asks whether the worker is employed and, if nonemployed, whether he or she is engaged in active job search activities (i.e., unemployed) during the preceding month. While the CPS is designed to provide a snapshot of the U.S. labor market for each month, one can use its longitudinal component to obtain measures of worker flows. We use the series constructed by the BLS.[6] Worker transition rates between employment and unemployment are, respectively, measured by:

$$\frac{EU_t}{E_{t-1}} \text{ and } \frac{UE_t}{U_{t-1}}, \tag{1}$$

where EU_t (UE_t) refers to the number of workers who switch their labor market status from "employed" ("unemployed") to "unemployed" ("employed") between month $t-1$ and t. EU_t and UE_t represent separations into unemployment and hires from unemployment, respectively. The definitions in Equation (1) give the EU transition rate and UE transition rate, respectively. The sample period for the BLS data is from January 1990 to December 2011.[7]

We also consider the job-to-job transition rate. Measuring job-to-job transitions in the CPS became feasible after the CPS redesign in 1994. Specifically, the dependent coding, which asks the individual if he or she is currently employed by the same employer as in the previous month, made it possible to measure the number of job-to-job movers. Fallick and Fleischman (2004) are the first to exploit this data structure for measuring the job-to-job

[4]The BED series are available at www.bls.gov/bdm.

[5]More precisely, average employment between the beginning and the end of the quarter is used for normalization.

[6]The data are available at www.bls.gov/cps/cps_flows.htm. Fujita and Ramey (2006) also construct flow series that are comparable to the BLS series. The cyclicality of the two data sets is very similar. See Fujita and Ramey (2006) for the data construction details and measurement issues in the CPS.

[7]In the BLS data, the flow that occurs from $t-1$ and t is dated at t. Using that convention, the BLS flow data start at February 1990.

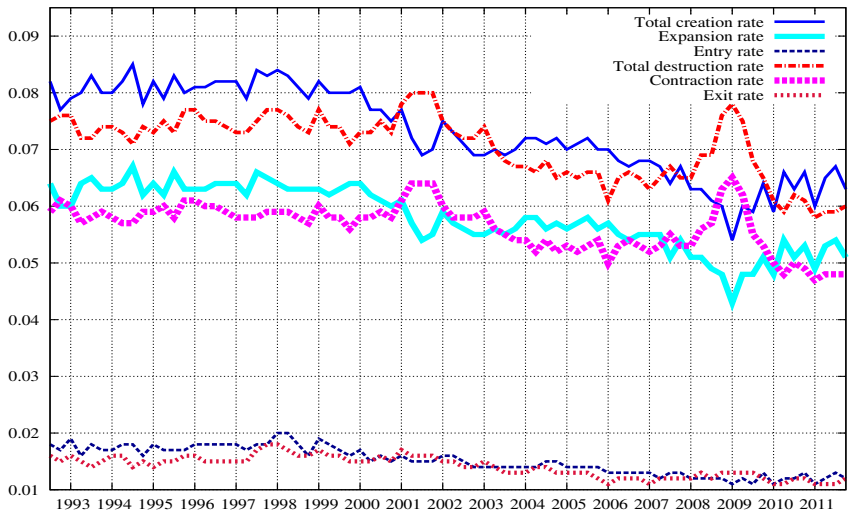

Figure 1: Job Creation and Destruction Rates

Notes: The data are taken from the BLS Business Employment Dynamics and cover the private business sector.

transition rate in the CPS.[8] Denoting the number of workers who are employed at different employers between $t-1$ and t by EE_t, we can write the job-to-job transition rate as

$$\frac{EE_t}{E_{t-1}}. \tag{2}$$

The Fallick-Fleischman data are updated regularly and the sample period for our analysis is from January 1994 to December 2011. All monthly transition rates are converted into quarterly series by time averaging.

2.2 Business Cycle Statistics

Unimportance of entry and exit. First, consider Figure 1, where we plot the time series of job flow rates. The figure shows not only the total rates of job creation and destruction but also their breakdowns into expansion, entry, contraction, and exit. The intention is to show the unimportance of the extensive margins for the business cycle fluctuations of job flow rates. According to the data, roughly 75% of total job flows come from expansion or contraction of the existing establishments at a quarterly frequency. More important, cyclical fluctuations of job flow rates are mostly accounted for by expansion or contraction: The correlation between the total job creation (destruction) rate and the expansion (contraction) rate is higher than 0.95. It is important to recognize that these two facts do not imply the

[8] Moscarini and Thompson (2007) explore several measurement issues of the CPS-based job-to-job transition rate and correct some measurement issues that existed in Fallick and Fleischman (2004). While their adjustments alter the overall level of the job-to-job transition rate somewhat, the time-series behavior is not significantly affected. We thus use the readily available series by Fallick and Fleischman (2004).

Table 1: Business Cycle Statistics for Worker Transition Rates and Job Flow Rates

	Standard Deviation	Relative Standard Deviation	Correlation with Output
Worker transition rates			
EU transition rate	0.071	5.966	−0.840
EE transition rate	0.056	4.620	0.698
UE transition rate	0.080	6.731	0.860
Job flow rates			
Creation rate	0.036	3.060	0.447
Destruction rate	0.045	3.838	−0.450
Stocks			
Unemployment rate	0.116	9.712	−0.889
Vacancies	0.121	10.153	0.862

Notes: First column: standard deviation of logged and HP-filtered series with smoothing parameter of 1,600. Middle column: standard deviation of each variable relative to that of real GDP. Sample periods: worker transition rates between unemployment and employment: 1990Q1–2011Q4; job-to-job transition rate: 1994Q1–2011Q4; job flow rates: 1992Q3–2011Q4; unemployment and vacancies: 1990Q1–2011Q4. The sample period for real GDP is adjusted to match the sample period of each variable.

unimportance of entry and exit at a lower frequency.[9] However, Figure 1 establishes our point that extensive margins are not important at the quarterly frequency. We thus abstract away from the extensive margin in our model on the basis of the quarterly evidence presented in Figure 1.

Table 1 characterizes the cyclicality of worker transition rates and job flow rates using standard business cycle statistics. The original series are logged and then detrended using the HP filter with smoothing parameter of 1,600. As mentioned above, original worker transition rates are monthly series. We render them quarterly by simple averaging. The real GDP series is used as a cyclical indicator to gauge each variable's volatility and cyclicality. We can summarize the characteristics of the labor market flows as follows:

- The EU transition rate is countercyclical, while UE and EE (job-to-job) transition rates are procyclical.
- The UE transition rate is somewhat more volatile than the EU transition rate.[10]
- The job destruction rate is somewhat more volatile than the job creation rate.

[9]The frequency of the measurement is important because, at a quarterly frequency, entrants become incumbents after a quarter, but the same entrants measured at annual frequency become incumbents only after one year. Thus, the share of job creation and destruction accounted for by entrants and exits becomes larger when measured at a lower frequency.

[10]Shimer (2012) and Hall (2006) argue that the separation rate into unemployment is roughly constant over the business cycle. Elsby et al. (2009), Fujita and Ramey (2006, 2009), Yashiv (2007), and Fujita (2011) argue otherwise.

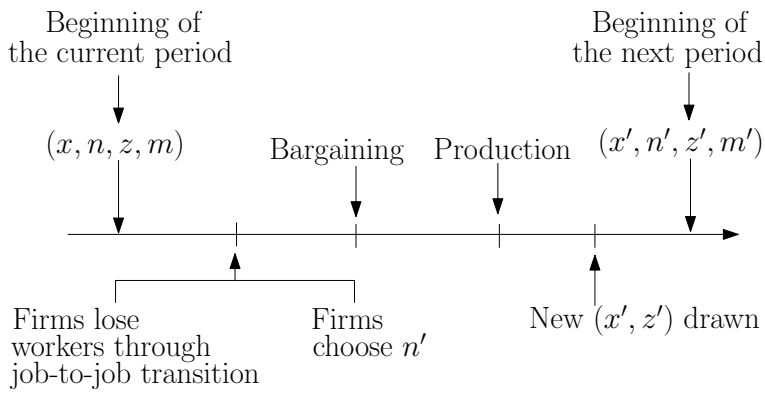

Figure 2: Timing of Events

- The job destruction rate is countercyclical and the job creation rate is procyclical. But the correlations with output are weaker in general than those of worker transition rates.

The fact that the correlations of job flow rates are weaker than those of worker transition rates is an important observation because it implies that those measures are not as robust as worker transition rates as business cycle indicators. We show that the same pattern holds in our model as well and we explore the underlying reasons for this observation.

Table 1 also shows volatilities of the unemployment rate and vacancies. As is well known in the literature, these two variables are quite volatile when compared with the volatility of labor productivity. The same is true with respect to output volatility. Lastly, a well-known fact about the cyclicality of unemployment and vacancies, i.e., the Beveridge curve, can also be observed from each variable's correlation with output.

3 Model

Our model is similar to the ones developed by Cooper et al. (2007) and Elsby and Michaels (2013). Time is discrete. There are two types of agents: firms and workers. Both are infinitely lived and risk neutral. The total measure of firms is normalized to one. The total measure of workers is denoted by L.

3.1 Timing

The timing of events is summarized in Figure 2. At the beginning of each period, a firm's idiosyncratic states are characterized by (x, n), where x represents idiosyncratic productivity and n represents the number of its workers. In addition, there is aggregate uncertainty in the economy in the form of a shock to aggregate productivity z. As will be clear later, each firm's decision is also influenced by the economy wide joint distribution of x and n and is written as $m(x, n)$. We summarize the aggregate states by $\mathbf{s} = \{z, m\}$. The stochastic processes for

z and x are, respectively, denoted by $G_z(z'|z)$ and $G_x(x'|x)$. Since we formulate the model recursively, we drop the time subscript from all variables and follow the convention that a primed variable denotes the variable at the beginning of the next period. Note, however, as indicated in Figure 2, the firm enters into the current period with the employment level n and produces with n' after labor turnover is completed in the current period. It then starts the next period with n'.

After the realization of productivities, firms make the separation/hiring decision. The hiring decision is subject to a search friction, which is discussed below. Hires include those from other firms (job-to-job transitions) as well as those from unemployment. Similarly, separations include worker flows to other hiring firms and to unemployment. As described in Figure 2, worker turnover occurs within a period. For example, vacancies posted at the beginning of the period after the realizations of productivities can be filled in the same period before production. After all worker flows are completed, wage negotiations between the employer and employees take place and then the firm produces.

3.2 Technology and Wages

The following decreasing-returns-to-scale production technology is available to all firms:

$$y = zxn'^{\alpha}. \tag{3}$$

Again, n' represents the number of workers who engage in production at the firm, whose beginning-of-the-period employment level was n. We make an important assumption regarding wage determination, following Cooper et al. (2007), that worker bargaining power is zero and thus all workers obtain a flow payoff of b. We study the equilibrium with a tie-breaking rule that the indifference implies taking a job if the worker is unemployed and taking a new job offer if the worker is employed. This assumption dramatically simplifies our analysis especially in the presence of on-the-job search. When wages depend on the state variables, the firm's hiring/firing decision and the workers' job acceptance decision will depend on the wage distribution of all firms and the wage offer distribution of hiring firms.[11] Note also that our implicit assumption here is the lack of commitment on the wage payment by the firm. In particular, the firm cannot promise to pay higher wages by asking the workers not to engage in on-the-job search. This can be supported by our timing assumption that wage is paid after the labor turnover including job-to-job transitions is completed every period. (The firm would renege the promise of paying more at the wage payment stage and thus the promise is not credible).

3.3 Search and Matching

Because of the search friction, only a fraction of job openings are filled every period. There is a flow vacancy posting cost, as in the standard model. The meeting technology takes the

[11] More precisely, we need the distributions of the marginal values of working and also of hiring firms. These distributions are even more difficult to compute than computing the wage and wage-offer distributions. See the earlier version of our paper (Fujita and Nakajima (2013) for more details on this issue.

following Cobb-Douglas form:

$$M = \mu S^{\psi} V^{1-\psi},$$

where S is the efficiency-weighted number of job seekers, V is the aggregate number of job openings, and μ is a scaling parameter. We normalize search efficiency of each unemployed worker at 1 and assume that each on-the-job seeker searches for a job at a reduced efficiency of $\gamma \in [0, 1]$. This specification allows us to abstract away from the search decision of the employed workers, while giving us the flexibility of matching the volume of job-to-job transitions in our quantitative exercise.

Recall that L is a fixed measure of the labor force. Thus, S can be written as

$$S = \gamma L(1 - U) + LU,$$

where U is the unemployment rate. Given the meeting technology, the contact probability for each vacancy posted is written as

$$q(\theta) = \frac{\mu S^{\psi} V^{1-\psi}}{V} = \mu \theta^{-\psi},$$

where $\theta = \frac{V}{S}$ is labor market tightness in this economy. Similarly for workers, the contact probability per unit of search is written as

$$f(\theta) = \frac{\mu S^{\psi} V^{1-\psi}}{S} = \mu \theta^{1-\psi}.$$

While unemployed workers meet a potential employer with this probability each period, the contact probability of employed workers, denoted by $f_e(\theta)$, is reduced by a factor γ as in

$$f_e(\theta) = \gamma f(\theta).$$

3.4 Optimal Employment Decision

The firm makes hiring and separation decisions by maximizing the present discounted value of its flow profits:

$$\Pi(x, n, \mathbf{s}) = \max_{n' \geq 0} \left\{ zxn'^{\alpha} - wn' - \mathbb{I}_{n' > (1-f_e(\theta))n} \kappa v + \beta \int \int \Pi(x', n', \mathbf{s}') dG_x(x'|x) dG_z(z'|z) \right\}, \tag{4}$$

under the forecasting functions $m' = \Phi_m(\mathbf{s})$ and $\theta = \Phi_\theta(\mathbf{s})$, where $\mathbf{s}$ represents a list of aggregate state variables, namely, $\{z, m\}$ as noted above (remember that m is the firm-type distribution). In Equation (4), v represents the number of vacancies posted and κ is the flow vacancy posting cost. The firm uses $\Phi_m(\mathbf{s})$ to evaluate the value in the next period and $\theta = \Phi_\theta(\mathbf{s})$ to forecast the quit (job-to-job transition) rate $f_e(\theta)$. Note that

$$v = \frac{n' - (1 - f_e(\theta))n}{q(\theta)}.$$

Observe that the firm loses $f_e(\theta)n$ workers through job-to-job transitions. As in Elsby and Michaels (2013), the optimal employment decision of the firm is characterized by an (s, S) rule, with the inaction region $(\underline{n}^*, \overline{n}^*)$ characterized by the following first-order conditions:

$$\alpha z x \underline{n}^{*\alpha-1} - w - \frac{\kappa}{q(\theta)} + \beta \int \int \Pi_n(x', \underline{n}^*, \mathbf{s}') dG_x(x'|x) dG_z(z'|z) = 0, \tag{5}$$

$$\alpha z x \overline{n}^{*\alpha-1} - w + \beta \int \int \Pi_n(x', \overline{n}^*, \mathbf{s}') dG_x(x'|x) dG_z(z'|z) = 0. \tag{6}$$

The envelope conditions are written as

$$\Pi_n(x, n, \mathbf{s}) = \begin{cases} \kappa \frac{1-f_e(\theta))}{q(\theta)} & \text{if } \tilde{n} < \underline{n}^*, \\ (1 - f_e(\theta)) \Big[\alpha z x n'^{\alpha-1} - w + \beta \int \int \Pi_n(x', \tilde{n}, \mathbf{s}') dG_x dG_z \Big] & \text{if } \tilde{n} \in [\underline{n}^*, \overline{n}^*], \\ 0 & \text{if } \tilde{n} > \overline{n}^*, \end{cases} \tag{7}$$

where $\tilde{n} = (1 - f_e(\theta))n$. Relative to Cooper et al. (2007) and Elsby and Michaels (2013), the key difference is that the pace of the job-to-job transitions at the firm depends on endogenous aggregate state variable θ. When $f_e(\theta)$ is high, the firm knows that more workers will leave the firm. Thus, for example, if the firm would like to expand its employment size, it needs to hire more workers to expand its size on net.

3.5 Equilibrium

The equilibrium of the model economy with the aggregate shock consists of the value function $\Pi(x, n, \mathbf{s})$, the employment decision rule $g(x, n, \mathbf{s})$, the wage function $w = b$, and the forecasting functions for the type distribution $m' = \Phi_m(\mathbf{s})$ and labor market tightness $\theta = \Phi_\theta(\mathbf{s})$, such that (i) $g(x, n, \mathbf{s})$ maximizes the value of the firm and $\Pi(x, n, \mathbf{s})$ is the associated optimal value function; and (ii) the forecasting functions $\Phi_m(\mathbf{s})$ and $\Phi_\theta(\mathbf{s})$ are consistent with the optimal employment decision of the individual firms. Note that Φ_m and Φ_θ are unknown functions of the aggregate state variables including the type distribution m. We capture the information in the distribution by the mean as discussed below.

4 Computation

The details of the computational algorithms are presented in the Appendix. To ease the notation, let us define the expected marginal profit function after the employment decision is completed in the current period as

$$D(x, n', z, m') = \int \int \Pi_n(x', n', z', m') dG_x(x'|x) dG_z(z'|z). \tag{8}$$

To approximate this function, we replace the type distribution m by the aggregate unemployment rate U.[12] The idea is the same as the solution technique used to solve heterogeneous

[12] Note that we approximate the expected value function $D(.)$ instead of $\Pi_n(.)$ since the former is smoother than the latter.

agent models with the uninsurable income risk, where the information in the wealth distribution is captured well with its mean. The D function is approximated by a piecewise linear function of the continuous state variables n', z, and U for each discretized value of idiosyncratic productivity x.

4.1 Steady-State Equilibrium

In the steady-state equilibrium, the aggregate state variable $\mathbf{s}$ is time-invariant and thus can be dropped. The first stage to solve for this equilibrium is to iterate on $\overline{n}^*(x,n)$, $\underline{n}^*(x,n)$, and $D(x,n')$ for a given guess of market tightness θ, using the first-order conditions (5) and (6).[13]

Once we obtain the convergence on the optimal employment adjustment function $n' = g(x,n)$ and the $D(x,n')$ function, the second stage of the algorithm simulates the economy to obtain the invariant distribution of $m(x,n)$. Using this invariant type distribution, we obtain the aggregate labor market variables, such as vacancies posted, the number of job seekers, and thus market tightness. The entire process repeats until the convergence on the employment policy function, the expected marginal profit function $D(x,n')$, and market tightness is completed.

4.2 Dynamic Stochastic Equilibrium

In the presence of aggregate uncertainty, current-period market tightness θ depends not only on realized aggregate productivity z but also on the type distribution $m(x,n)$. In making the employment adjustment decision, each firm therefore needs to know the relationship between θ and $m(x,n)$ as well as z. As mentioned above, it is assumed that the firms use only the mean of the distribution (aggregate employment and thus equivalently unemployment U) to summarize the information in $m(x,n)$, following the methodology often used in models of uninsurable income risk (e.g., Krusell and Smith (1998)). Further, calculating and updating the $D(x,n',z,m')$ function requires the firms to form the forecast for the next-period type distribution. Given our assumption about the approximate equilibrium, this entails forecasting next-period aggregate unemployment, U', using current-period unemployment and realized aggregate productivity.

The algorithm starts with guessing a set of coefficients of the forecasting rules. Given these rules, we can solve for each individual firm's problem, following the procedure used to solve for the steady-state equilibrium (except that those functions now depend on the aggregate state variables). Once we achieve the convergence on the $D(x,n',z,U')$ function and the employment policy function $g(x,n,z,U)$, we simulate a large panel data set from which we can obtain a long time series of $\{z,U,\theta\}$. By using these objects, we can update the forecasting rules by running OLS regressions. The algorithm stops when the convergence on the coefficients of the forecasting rules is achieved.

[13] Strictly speaking, $\overline{n}^*(x,n)$ actually does not depend on n, but we are using more general notation here.

Table 2: Model Parameters

Symbol	Description
ψ	Elasticity of matching function with respect to job seekers
α	Curvature of the production function
β	Time discount factor
μ	Scale parameter of the matching function
κ	Vacancy posting cost
γ	Search intensity of on-the-job seekers
b	Flow outside benefit
ρ_x	Persistence of the idiosyncratic productivity process
σ_x	Standard deviation of the idiosyncratic shock
ρ_z	Persistence of the aggregate productivity process
σ_z	Standard deviation of the aggregate shock
L	Labor force (population) size

5 Calibration

One period in the model is assumed to be one month. The exogenous productivity processes follow the standard AR(1) processes:

$$\ln z' = \rho_z \ln z + \varepsilon'_z,$$
$$\ln x' = \rho_x \ln x + \varepsilon'_x,$$

where $\varepsilon_x \sim N(0, \sigma_x^2)$ and $\varepsilon_z \sim N(0, \sigma_z^2)$. These processes are then approximated by a finite-state, first-order Markov chain.[14]

Note that some of the statistics used to calibrate the model are available only at quarterly frequency or annual frequency. In particular, job flow rates are measured by taking net employment changes over a quarterly period and it is important for us to construct the model-based statistics in the same way as in the observed data. The details are discussed below.

We partition the model parameters into two groups: the one determined exogenously to the model without solving the model and the other determined by matching the empirical moments. Table 2 provides the summary of the model parameters.

5.1 Parameters Set Exogenously

First, the time discount factor β is set to 0.996, which implies the quarterly discount factor of 0.99, a standard value used in the business cycle literature. The curvature of the production

[14] While we use the finite-state approximation in calculating the conditional expectation with respect to aggregate uncertainty (when we solve the firm's problem), we maintain the original AR(1) process in the simulation stage so that the process has a continuous state space. This enables us to generate the smooth impulse response functions presented below.

function α is set to 0.72. This appears to be a value commonly used in the literature, for example, by Cooper et al. (2007). We also considered a lower value (0.67) and found that the quantitative results were hardly affected. The elasticity of the matching function with respect to unemployment $1 - \psi$ are set to 0.5. The persistence parameter of aggregate productivity is set to 0.983, which implies a quarterly autocorrelation of 0.95, following the convention of the business cycle literature. The persistence parameter of idiosyncratic productivity is set to 0.95. It is chosen to be fairly persistent following the literature. We also experimented with different values (such as 0.99 and 0.90) and found that the results were insensitive with respect to these alternative values.

5.2 Parameters Set Endogenously

The remaining seven parameters are set by matching the seven moment conditions. For the purpose of clarity, we describes the procedure by associating one particular parameter with one moment condition that is most useful for the identification. In truth, however, each parameter cannot be set separately and thus our procedure should be considered as jointly minimizing the distance between the model-based moments and corresponding empirical moments.

First, we use the scale parameter of the matching function μ and the vacancy posting cost κ to achieve the average levels of the monthly UE transition rate $f(\theta)$ and job filling rate $q(\theta)$ at 0.25 and 0.9, respectively. The former number is based on the time-series data on the UE transition rate computed from the CPS labor flow data for the period January 1990 through December 2011.[15] The latter is based on the evidence by Davis et al. (2013), who show that the daily job filling rate fluctuates at around 7%, which translates into the monthly filling rate of 0.9.[16] We set μ at 0.474 and κ at 0.269 to hit these targets as closely as possible.

The search intensity parameter of employed workers γ is selected to match the average job-to-job transition rate in the Fallick and Fleischman (2004) data that cover the period between January 1994 and December 2011. The average job-to-job transition rate over this period is around 2.5% in the data. Remember that we target the monthly UE transition rate $f(\theta)$ at 0.25. Thus, $\gamma = 0.1$ allows us to match the job-to-job transition rate of 2.5%.

Next, we calibrate the standard deviation of the shock to firm-level productivity σ_x by referring to the dispersion of the employment growth distribution. Davis et al. (2007) calculate employment-weighted cross-sectional dispersion (standard deviation) of annual employment growth rates using the Longitudinal Business Database (LBD) for the period 1978 through 2001. Their figure shows that the standard deviation fluctuates roughly around 0.60. Our model generates a value close to this target with $\sigma_x = 0.078$. Note that the empirical measure is based on net employment changes over an annual interval. We therefore construct the corresponding statistic in our model, taking net employment changes over 12 periods.

[15] Given that all employed workers accept outside offers (conditional on searching on the job, which is exogenously set by γ), the job finding rate of on-the-job seekers is identical to the one for unemployed workers.

[16] Fujita and Ramey (2012) also use the same target value, based on the evidence by Barron et al. (1997).

Table 3: Summary of Calibrations

	Exogenously set						
	ψ	α	β	ρ_z	ρ_x		
Benchmark	0.5	0.72	0.996	0.983	0.95		
$\gamma = 0$	same	same	same	same	same		
	Endogenously chosen						
	μ	κ	L	b	γ	σ_x	σ_z
Benchmark	0.474	0.269	15.894	0.349	0.100	0.078	0.003
$\gamma = 0$	same	0.621	15.900	0.346	0.000	same	same

Next, the labor-force size L is chosen to achieve the average establishment size of 15. Henly and Sanhez (2009) show that the mean establishment size in the U.S. has been stable around 15 between 1974 and 2006. The mass of firms is normalized to one in our model. Thus, the steady-state mean establishment size of 15 also implies the steady-state employment stock of 15. Together with the steady-state unemployment rate of 5.6% (which is discussed next), we can set $L = 15.89$.[17]

The flow outside option value b is set to 0.349. Conditional on the values of the other parameters, this parameter can be used to hit the average monthly EU transition rate at 1.5% (and thus the unemployment rate, given the target level of the UE transition rate, as discussed above). Note that given our assumption of zero worker bargaining power, this value is equal to wage and thus has a strong influence on the employment decision of the firm. Note that although we discussed the calibration of b by linking it the EU transition rate, this parameter in truth should be considered as another parameter that influences the firm's hiring and firing decision jointly with the other parameters such as the vacancy posting cost κ and the scale parameter of the matching function μ.

There has been a contentious debate about the appropriate level of b (Costain and Reiter (2008) and Shimer (2005) versus Hagedorn and Manovskii (2008)). Elsby and Michaels (2013) show that, in the environment with decreasing-returns-to-scale production technology, their model can generate larger volatility for a given level of the ratio between b and average labor productivity. In our calibration, the value of b (and thus w) amounts to 70% of average labor productivity. One important point to note here is that, under our assumption of zero worker bargaining power, wage is completely rigid at b. We discuss its quantitative implications below.

Lastly, the size of the aggregate shock σ_z is set to 0.003 and selected by matching the standard deviation of the aggregate output series. Over the sample period from 1990 to 2011, the standard deviation of the logged and HP-filtered real GDP series is 0.012.[18] The

[17] In the standard model with one-worker firms, the labor-force size is often normalized to one and the steady-state employment stock corresponds to the value equal to one minus the unemployment rate. The same idea applies here, but we use the average establishment size to pin down the labor-force size.

[18] We use the post-1990 output series simply because other series we use are available from (or shortly after) 1990. See the notes to Table 1.

chosen value allows us to roughly match this level of volatility.

5.3 Model Without Job-to-Job Transitions

With the assumption of $\gamma = 0$, our benchmark model reduces to the model without job-to-job transitions. In calibrating this simplified model, it is important to ensure that the comparison between the two models is quantitatively fair. First, we set parameter values that are exogenously chosen in the benchmark model to the same values. We also keep the same process for the aggregate TFP process, leaving us to set the five remaining parameters (σ_x, μ, κ, L, and b). First, we keep the same values for σ_x. This implies that we drop the moment condition for the employment growth dispersion of 0.6. We find this to be more prudent for the purpose of comparing the quantitative properties of the two models. The reason is that matching the level of the employment growth dispersion at the target level requires raising σ_x significantly, which in itself has an effect of lowering the overall volatility of the model.[19] Keeping the same idiosyncratic productivity process allows us to shut down the effects coming through the different levels of uncertainty facing the firms. The remaining four parameters are set by matching the same four moment conditions discussed above, namely, $f(\theta) = 0.25$, $q(\theta) = 0.9$, the average employment size of 15, and the EU transition rate of 0.015. This further implies that the value of μ remain the same as before, and the values of L, b, and κ are set to different values. We find that this procedure yields a higher value of the vacancy posting cost κ, but the value of b remains very close. In particular, the level of b relative to the average productivity in this simpler model is 0.696.

6 Main Results

This section presents the main results of the paper. We first show that the model matches the first moments of the observed data reasonably well. We also make sure that the model is capable of replicating the "hockey stick" hiring and separation functions recently studied by Davis et al. (2012). We then examine if the model can also replicate the cyclicality of worker transition rates and job flows rates.

6.1 Steady-State Properties

The first-moment properties of the model are summarized in Table 4. Under the benchmark calibration, we match almost exactly the moment conditions we targeted. The table also presents how well the model does in matching the average job flow rate, which is not explicitly targeted in our calibration. We can see that the benchmark model does fairly well in replicating the level of job turnover at the quarterly frequency (9.1% in the model versus 8% in the data).

[19] We actually considered the case with a higher value of σ_x and found that it significantly lowers the volatility of the model.

Table 4: Steady-State Properties

	Data Collection Frequency	Model Benchmark	$\gamma = 0$	Empirical Target
Worker-side data				
EU transition rate	Monthly	0.015	0.015	0.015
EE transition rate	Monthly	0.025	–	0.025
UE transition rate	Monthly	0.250	0.250	0.250
Unemployment rate	Monthly	0.057	0.057	0.057
Establishment-side data				
Job creation/destruction rate	Quarterly	0.091	0.045	0.080
Average establishment size	–	14.98	14.99	15.00
Employment growth dispersion	Annual	0.600	0.450	0.600
b/(average labor productivity)	–	0.695	0.696	–

Turning to the model without job-to-job transitions, we can see that the model has no trouble matching the levels of EU and UE transition rates, and thus the level of the unemployment rate. As we discussed in the previous section, the model does not generate enough employment growth dispersion, given that we did not recalibrate the level of idiosyncratic uncertainty.[20] The main problem of the model without job-to-job transitions, however, is that it fails to match the average level of the job flow rate. The job destruction rate in this model (4.5% at the quarterly frequency) is very close to the sum of the EU transition rate (1.5% at the monthly frequency) over a three-month period, even though the job destruction rate is constructed as a sum of *net* employment changes at shrinking establishments over a quarter (normalized by the stock of employment). In the benchmark model, on the other hand, both EU separations and job-to-job separations at those shrinking establishments constitute underlying job destruction. We will come back to this discussion below.

In the last row, we also report the ratio of the flow outside option value to average labor productivity. Remember that this ratio in our calibration is not targeted at a particular value. Nevertheless, it is close to the value often used in the literature. Note also that two models share a similar value. In relation to Elsby and Michaels (2013), there are a few things to keep in mind (apart from the fact that their model does not feature job-to-job transitions). First, this value is 0.61 in their paper, and thus, the higher values in our models are likely to contribute to raising the volatilities. Second, our model is different from theirs, even when no job-to-job transitions are allowed, due to our assumption of no worker bargaining power (and thus completely rigid wage). This assumption also implies a larger magnification of the aggregate shock in our models. Thus, we do not claim that we solve the so-called Shimer puzzle. Note, however, that we can still make a fair comparison between the models with and without job-to-job transitions given that both models share the same wage setting.

[20] As discussed in footnote 19, this is a deliberate choice to make the comparison of the quantitative properties of the two models as fair as possible.

6.2 Hockey Stick Functions

A paper by Davis et al. (2012) characterizes the cross-sectional relationships between hiring/separation rates and job flow rates, using the so-called hockey stick functions. Here, we show that our benchmark model replicates the main features of the well-established empirical relationships, using the simulated observations from the steady-state version of our model.[21] In Panel (a) of Figure 3, the horizontal axis measures net employment growth over a quarterly period. Thus, the firms located below (above) zero are destroying (creating) jobs over the quarterly period. The vertical axis measures the total hiring and separation rates over the same quarterly period. Note that in our model, the total hiring (or separation) rate corresponds to the sum of hires from (or separations into) unemployment and other firms (job-to-job transitions), normalized by the employment stock. The quarterly measures add up all hires (separations) that occur during the quarterly period. The circles in the scatter plot indicate the relationship between the hiring rate and net employment growth at individual firms, while the triangles show the relationship between the separation rate and net employment growth. Blue solid and red dashed lines represent approximations of these relationships obtained from the cubic polynomial regressions. Panel (b) plots the same relationships at the frequency of the model (i.e., monthly) and thus removes the effect of time aggregation that is present at a quarterly frequency. The empirical result by Davis et al. (2012) measures net employment growth at a quarterly frequency and thus corresponds to the result in Panel (a).

This figure provides an important insight into why hiring/separation rates and job creation/destruction rates can be different from each other. One can clearly see on the left-hand side of Panel (a) that there are some firms that are shrinking on net (destroying jobs) at a quarterly frequency, yet hiring workers, represented by the circles. Observe, however, that hiring rates at the firms that are shrinking at a rapid pace are normally zero. Further, when the data are sampled at a monthly frequency, the circles that indicated positive hiring rates at shrinking firms in Panel (a) disappear, implying that separation and job destruction rates at those shrinking firms are nearly equivalent to each other at a monthly frequency (note that they are only "nearly" equivalent for the reason discussed below). In other words, positive hiring rates that are observed at the shrinking firms are due to time aggregation.[22] In the empirical relationship reported by Davis et al. (2012), there are many shrinking establishments that are hiring on a gross basis, and the extent of this happening is clearly more significant than can be seen in our simulated quarterly data. For example, Figure 6 in their paper shows that, even at the firms that are cutting employment by 60%, the gross hiring rate is higher than 10%. This type of pattern simply cannot be explained by time aggregation only.[23]

[21] Our calculation is based on a random sample of 10,000 establishments (of all 1 million establishments) over a quarterly period in the steady-state equilibrium.

[22] There are many logical possibilities as to why positive hires are observed at firms that are shrinking on net at a quarterly frequency. For example, a firm hires some workers in the first month but lays off workers in the following two months due to negative shocks and, thus, shrinks on net.

[23] As discussed by Davis et al. (2012), nontrivial positive hiring at net shrinking firms in the actual data

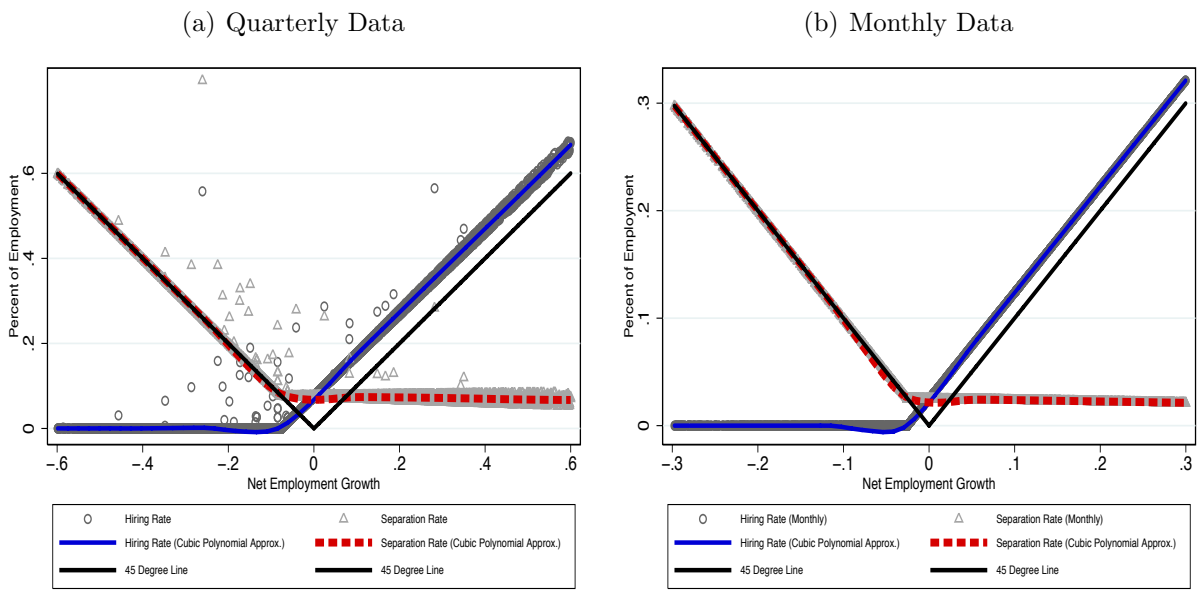

Figure 3: Hiring/Separation Rate as a Function of Net Employment Growth Rate

Notes: Based on the 10,000 simulated firm-level observations over quarterly (Panel (a)) and monthly (Panel (b)) periods in the steady-state equilibrium. Blue-solid and red-dashed lines represent predictions from the cubic polynomial regressions.

Note that it is incorrect to say, however, that separations and job destruction are the same to the left of zero in the model (even at a monthly frequency). The equivalence holds only at the firms that are making relatively large employment adjustments. One can see in Panel (b) that there is a small region to the immediate left of zero where nonzero hires are observed. One can also see that, over the same small region, there are more separations than net employment changes (the red line is located above the 45 degree line). Although this region looks small, there are a large number of firms located in this region. For example, 68% of firms are located in the region between 0 and -5% in the steady-state equilibrium. This phenomenon corresponds to the scenario in which the firm wants to reduce its employment (thus destroying jobs), but job-to-job transitions are more than enough to achieve the target employment level. Thus, these firms end up needing to hire at least some workers. In other words, when a firm loses its workers through job-to-job transitions (which generate gross separations), the firm finds it optimal to partially replace these workers. In this case, hires and separations coexist within a period, and employment growth is negative. This case is unlikely to occur when a firm receives a large negative shock, in which case the firm is willing to cut its workforce beyond job-to-job transitions.

Next, consider the firms that are located to the right of zero net employment growth (thus

is likely because of some essential positions that the firm needs to refill (after workers leave the firm), even though firm size is rapidly shrinking on net. Such a feature is simply not present in our model.

creating jobs). One can see in Panel (a) that these firms experience a nontrivial number of separations even when they are expanding their employment at a rapid pace. The same is true even at the monthly frequency displayed in Panel (b), and thus time aggregation is not the reason for the observed nontrivial separations. In the model, even at the firms that are growing rapidly, there are always workers leaving for other hiring firms. The pattern of separations spreading over the entire range of positive employment growth is consistent with the empirical fact presented by Davis et al. (2012).[24]

In summary, the conceptual differences between hiring/separation rates and job creation/destruction rates arise because (i) the firms that are destroying jobs on net may hire workers and (ii) separations occur at the firms that are creating jobs. The first difference exists only at the firms with small negative employment growth, whereas the second difference arises over entire positive employment growth rates.

Remember that this discussion applies to our benchmark model with job-to-job transitions. In the model without job-to-job transitions, neither (i) nor (ii) are true in the model frequency. Although time aggregation can generate those features, its effects are very small. Note also that introducing exogenous separations into unemployment easily makes it possible to produce these two patterns. However, the crucial point is that the pace of separations at hiring firms varies with business cycles in our model (because of the procyclicality of the job-to-job transition rate), whereas in the model with exogenous separations, it is invariant with respect to business cycles.

6.3 Cyclicality of Worker Transition Rates

Table 5 presents the same second-moment statistics discussed earlier in Table 1. As in the previous analysis, we compare properties of the two models with and without job-to-job transitions. Let us first discuss the model without job-to-job transitions (the case with $\gamma = 0$). In terms of relative volatilities of UE and EU transition rates, the model does a decent job compared with those of the observed series presented in Table 1, except for several problems. The EU transition rate is too volatile relative to the data; and the volatility of the EU transition (separation) rate is higher than that of the UE transition (job finding) rate, although the opposite is true in the data.

Overall volatility of this model is large despite the relatively low level of the ratio between flow outside option b and average labor productivity (0.70). Here, the fixed wage specification plays an important role for the large response of both the separation rate and the job finding rate. When a positive aggregate shock hits the economy, hiring firms expand employment more under the fixed wage environment than in the flexible wage environment (because wages do not rise in this environment). Similarly in a recession, contracting firms respond more in shedding workers because wages do not fall in the fixed wage environment. The correlation patterns of these two transition rates are largely in line with the data.

[24]Note, however, that in our model, the separation function is flat on the right-hand side of zero. It is more plausible that the separation function is downward sloping in that region, because workers are less likely to leave more rapidly growing firms. Our model does not generate such a pattern because workers receive the same wage regardless of the pace of employment growth.

Table 5: Business Cycle Statistics: Models With and Without Job-to-Job Transitions

	Standard Deviation		Relative SD		Corr. with Output	
	Benchmark	$\gamma = 0$	Benchmark	$\gamma = 0$	Benchmark	$\gamma = 0$
Worker transition rates						
EU transition rate	0.131	0.100	10.93	9.33	−0.85	−0.75
EE transition rate	0.063	0.000	5.21	0.00	0.99	0.00
UE transition rate	0.063	0.074	5.21	6.87	0.99	0.98
Job flow rates						
Creation rate	0.023	0.070	1.90	6.52	−0.19	−0.71
Destruction rate	0.033	0.101	2.76	9.36	−0.48	−0.74
Stocks						
Unemployment rate	0.157	0.130	13.05	12.06	−0.89	−0.89
Vacancies	0.085	0.049	7.11	4.59	0.94	0.60

Notes: Based on the simulation of a panel of 1 million establishments over 1,200 (monthly) periods. Worker flows, worker transition rates, unemployment rate, and vacancies are converted into quarterly data by time averaging. Job flows are based on net employment changes over a quarter. All observations are logged and HP filtered with smoothing parameter of 1,600.

Our benchmark model performs similarly in terms of EU and UE transition rates and it also generates procyclical and fairly volatile job-to-job (EE) transition rate. However, more important differences between the two models arise when job flow rates are considered.

6.4 Cyclicality of Job Flow Rates

The middle rows of Table 5 present the cyclical statistics of job flow rates. Again, let us first discuss the performance of the model without job-to-job transitions. Overall, this model performs poorly in replicating the cyclical properties of job flow rates (see Table 1). First, the volatility of job flow rates is much larger in this model. Second, in the data, the job creation rate is procyclical and the job destruction is countercyclical, whereas in the model, both are strongly countercyclical. Panel (b) of Figure 4 presents the impulse response functions of job flow rates to a one-standard-deviation negative aggregate shock in this model. Note that the figure plots the impulse responses at the model frequency (monthly) since we want to focus here on the underlying mechanism without the effects of time aggregation. The source of the strong countercyclicality of the job destruction rate is clear: The negative shock results in a large spike in the job destruction rate. Although the negative shock causes the job creation rate to fall initially, it bounces back quickly, staying above its steady-state level for a long period of time. Note that in this model, the hiring flow from unemployment is the only source of job creation and that the job creation rate is defined as this flow normalized by the employment stock. The flow from the unemployment pool is countercyclical because the unemployment pool itself is countercyclical even though the UE transition rate is strongly procyclical. We will come back to his point below again.

Relative to this model, our benchmark model with job-to-job transitions does a much

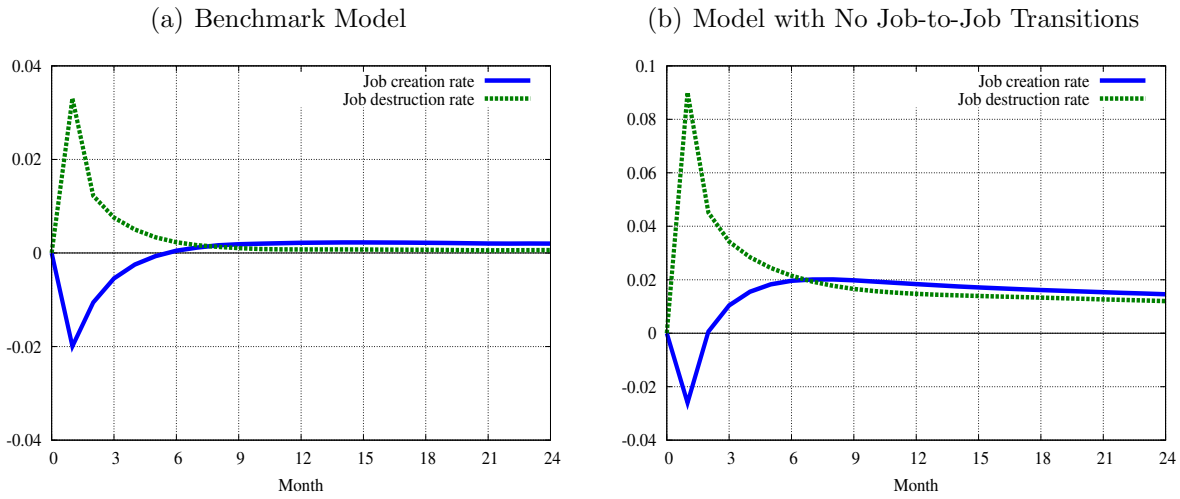

Figure 4: Impulse Response Functions of Job Flow Rates

Notes: Plotted are responses to a one-standard-deviation negative aggregate shock expressed as log deviations from the steady-state levels.

better job of replicating the observed pattern of job flow rates. Their responses to the shock are much more muted in line with the observed data (i.e., volatilities of the two variables are notably reduced); although the job creation rate remains countercyclical, the countercyclicality weakened considerably. In Section 6.8, we consider an extension to partially resolve this problem.

Panel (a) of Figure 4 plots the impulse response functions of job creation and job destruction rates in our benchmark model. One can see that the initial increase in the job destruction rate is much smaller and returns close to its steady-state level quickly, in contrast to the previous case in which the destruction rate stayed significantly higher above its steady-state level for an extended period of time. Furthermore, the job creation rate, while its initial response is similar to the one in the model without job-to-job transitions, does not exhibit the "overshooting" behavior in the previous model, which was the source of its strong countercyclicality. It is clear that the presence of job-to-job transitions in our model contributes to significant improvements in both volatility and the correlation pattern.

6.5 Linking Worker Transition Rates and Job Flow Rates

We now explore more explicitly the link between worker transition rates and job flow rates in our benchmark model. In Table 6, we present the second-moment statistics of various flow measures, splitting them into the creation/hiring measures and destruction/separation measures. The first two rows in each block compare the cyclicality of quarterly and monthly measures of job flow rates. Recall that the empirical measures of job flow rates correspond to the quarterly ones and are based on establishment-level net employment changes over a

Table 6: Job Flow Rates and Worker Transition Rates

		Standard Deviation	Relative Standard Deviation	Correlation with Output
Job creation rate	(Q)	0.023	1.91	−0.19
Job creation rate	(M)	0.022	1.85	0.08
Total hiring rate	(M)	0.026	2.18	0.40
UE hiring rate	(M)	0.111	9.26	−0.77
Job destruction rate	(Q)	0.033	2.76	−0.48
Job destruction rate	(M)	0.026	2.16	−0.33
Total separation rate	(M)	0.019	1.62	0.09
EU separation rate	(M)	0.131	10.93	−0.85

Notes: The letter in parentheses indicates the data collection frequency (quarterly or monthly). Monthly job flow rates are constructed by applying the same idea for the quarterly job flow rates to monthly employment changes: job creation (destruction) rate = sum of employment changes at expanding (shrinking) establishments over a monthly period (normalized by employment). Hiring rate: all hires (sum of UE and EE worker flows) as a fraction of employment. Separation rate: all separations (sum of EU and EE worker flows) as a fraction of employment.

quarterly period. Considering the monthly measures allows us to isolate the effects of time aggregation.

Job creation rate versus hiring rate. To understand the cyclical properties of the job creation rate, it is helpful to inspect its relationship with hiring rates. The third and fourth rows in Table 6 present the cyclical properties of the total hiring rate and UE hiring rates, respectively.[25] Note that the denominator of both of these variables is the beginning-of-the-period stock of employment. The first variable includes all hires (hires from other firms and hires from the unemployment pool), whereas the latter variable only considers the flow from unemployment. It is important to note that the "hiring rate" is different from the worker transition rate. The concept of the hiring rate is useful because it is scaled by the same variable (employment) as the job creation rate.[26]

One can see that the cyclical properties of the job creation rate differ significantly from those of the two hiring rates, even apart from the effects of time aggregation. To see the underlying reasons, first note that the total hiring rate can be written as:

$$h_t = \frac{\sum_i v_{it} q_t}{E_t} = \frac{V_t q_t}{E_t} = \frac{f_t(\gamma E_t + U_t)}{E_t} = \underbrace{f_t \gamma}_{\text{EE transition rate}} + \underbrace{f_t \frac{U_t}{E_t}}_{\text{UE hiring rate}}, \tag{9}$$

[25] The difference between these two hiring rates corresponds to the job-to-job transition rate, whose second-moment property is presented in Table 5.

[26] Strictly speaking, the job flow rates are normalized by the average employment stock in t and $t+1$, while the hiring rate is normalized by the beginning-of-the-period employment stock. This difference is immaterial for the cyclicality of these variables.

where V_{it} is the number of vacancies posted at establishment i in t; V_t is the aggregate number of vacancies in t; E_t is the employment stock at the beginning of t; q_t and f_t are the job filling rate and the job finding rate, respectively, in t and should be understood as a function of market tightness θ_t. The two terms after the last equality are, respectively, the job-to-job (EE) transition rate and the hiring rate from the unemployment rate. Clearly, the job-to-job (EE) transition rate is procyclical as a direct effect of f_t. On the other hand, the hiring rate from unemployment is strongly countercyclical, as indicated by the fourth row of Table 6. This is because the strong countercyclicality of U_t/E_t dominates the procyclical effect of f_t. Note also that even though the volatility of the UE hiring rate is large, the total hiring rate is much less volatile. The reason is the negative correlation between the procyclical job-to-job transition rate and the countercyclical UE hiring rate. The total hiring rate turns to be procyclical because the volume of job-to-job hires is larger than that of UE hires and thus the procyclicality of the former variable is dominant.[27]

Table 6 also indicates interestingly that the total hiring rate is quite a different object from the job creation rate. In particular, although the procyclicality of the total hiring rate is relatively strong, this procyclicality nearly disappears when the job creation rate is considered. To see the source of this difference, let us write the job creation rate (at monthly frequency) Cre_t as follows:[28]

$$Cre_t = \frac{\sum_{\{i|\Delta E_{it}>0\}}(q_t v_{it} - f_t \gamma E_{it})}{E_t}. \tag{10}$$

This definition aggregates net employment changes at expanding firms. The first term on the right-hand side aggregates vacancies posted at those expanding firms only. In the model, there exist vacancies posted at firms that are shrinking on net. As discussed before, these vacancies are posted in order to replace some of the workers that are lost through job-to-job transitions (and thus these firms are shrinking on net). However, the vast majority of vacancies are posted at expanding firms. In the steady state, more than 98% of vacancies come from expanding firms.[29] This observation allows us to adopt the following useful approximation:

$$\sum_{\{i|\Delta E_{it}>0\}} v_{it} \approx V_t.$$

[27]Note that in our calibration, $\gamma = 0.1$ and $U/E = 0.06$ in the steady state. Thus, the cyclicality of the job-to-job transition rate carries more weight.

[28]Note that the following definition uses, for simplicity, the beginning-of-the-period stock of employment as a normalizing factor (instead of the average of the two periods as suggested by the original definition). However, this simplification bears no material implications for our quantitative analysis.

[29]This share obviously depends on calibration. However, given that the level of the job-to-job transition rate in the model is calibrated to the observed data, there is little room for this share to be different.

Under this approximation, Equation (10) can be rewritten in the following ways:

$$Cre_t \approx \frac{f_t[\gamma E_t + U_t] - \sum_{\{i|\Delta E_{it}>0\}} f_t \gamma E_{it}}{E_t} = h_t - f_t \gamma \frac{\sum_{\{i|\Delta E_{it}>0\}} E_{it}}{E_t} \tag{11}$$

$$= f_t \gamma \left(1 - \frac{\sum_{\{i|\Delta E_{it}>0\}} E_{it}}{E_t}\right) + f_t \frac{U_t}{E_t}. \tag{12}$$

Note that the term $\sum_{\{i|\Delta E_{it}>0\}} E_{it}/E_t$ in (11) gives the fraction of workers employed at expanding firms. Thus, the last term in (11) subtracts job-to-job transitions that occur *across expanding firms* from the total hiring rate. To obtain (12) from (11), we use the fact that the total hiring rate h_t is the sum of the job-to-job transition rate and the UE hiring rate as in (9). An important insight here is that as the share $\sum_{\{i|\Delta E_{it}>0\}} E_{it}/E_t$ increases, the job creation rate behaves more similarly to the UE hiring rate and thus becomes more countercyclical. The extreme (and unrealistic) case is when this share is one and the job creation rate is identical to the UE hiring rate. In this case, none of job-to-job transitions contribute to new job creation because all job-to-job transitions simply represent reshuffling of workers within the group of expanding firms. The opposite extreme (and again unrealistic) case is one in which this share is zero. In this case, all job-to-job hires contribute to job creation because they are all coming from shrinking firms, and thus the job creation rate is identical to the total hiring rate. The actual result in our benchmark model shows that the job creation rate (at the monthly frequency) is weakly procyclical. The discussion so far provides the important insight that the cyclicality of the job creation rate is bounded by the countercyclicality of the UE hiring rate and the procyclicality of the job-to-job transition rate.[30]

Recall that the model without job-to-job transitions performed much worse than our benchmark model in terms of replicating the cyclical pattern of the job creation rate: Its volatility was too high, and it exhibited very strong countercyclicality (see the discussion in Section 6.4). The discussion in this section makes it clear where these problems are coming from: In that model, the job creation rate is identical to the UE hiring rate at the model (monthly) frequency, and the UE hiring rate is much more volatile and strongly countercyclical. The presence of the procyclical job-to-job hiring rate in our model plays an important role for mitigating both of these problems.

However, our model is still short of generating the procyclical job creation rate compared with the actual data. However, in light of the above discussion, we can speculate how this issue can be addressed by extending our model. First, note that in our model, the "search intensity" of employed workers is fixed at γ. Endogenizing the search intensity is likely to make the job creation rate more procyclical. Second, recall also that our assumption about the wage setting makes the job acceptance decision irrelevant. However, when the job acceptance decision is endogenized, its procyclicality is also likely to contribute to a

[30] Of course, the share $\sum_{\{i|\Delta E_{it}>0\}} E_{it}/E_t$ varies over the business cycle, affecting the cyclicality of the job creation rate as well. The share actually behaves procyclically, thereby reducing the procyclicality of the job creation rate.

higher procyclicality of the total hiring rate and thus the job creation rate. Lastly, the more procyclical job finding rate f_t itself will be an effective remedy for this problem as well.

Job destruction rate versus separation rate. Next, a similar discussion can be applied to the difference between the job destruction rate and the separation rate. First, we can write the total separation rate as:

$$s_t = \frac{\sum_i l_{it} E_{it} + f_t \gamma E_{it}}{E_t} = \underbrace{EU_t}_{\text{EU transition rate}} + \underbrace{f_t \gamma}_{\text{EE transition rate}}, \tag{13}$$

where l_{it} is the layoff rate at establishment i. We use the term "layoff rate" here simply to represent the rate at which firms actively shed workers and all of these workers enter the unemployment pool. Thus, the first term after the first equality corresponds to the EU transition rate. The second term represents the job-to-job transition rate. The EU transition rate is countercyclical, while the latter term is procyclical. Overall, the total separation rate is largely acyclical due to the offsetting effects of the two terms. Next, consider the following definition of the job destruction rate Des_t:

$$Des_t = \frac{\sum_{\{i|\Delta E_{it}<0\}} (l_{it} E_{it} + f_t \gamma E_{it})}{E_t} = EU_t + f_t \gamma \frac{\sum_{\{i|\Delta E_{it}<0\}} E_{it}}{E_t}. \tag{14}$$

The only difference from the definition of the total separation rate is that the job destruction rate aggregates separations occurring at shrinking firms only. Note that $\sum_{\{i|\Delta E_{it}<0\}} l_{it} E_{it} = \sum_i l_{it} E_{it}$ because there are no firms that lay off workers while also expanding on net. Therefore, this selection of shrinking firms is irrelevant for the aggregation of separations into unemployment. This selection matters for the aggregation of job-to-job transitions, however. The expression after the second equality in Equation (14) shows that the job destruction rate equals the sum of the EU transition rate plus the job-to-job transition rate multiplied by the share of employment at shrinking firms. By comparing (13) and (14), one can see that the job destruction rate is more countercyclical than the total separation rate for two reasons. First, $\sum_{\{i|\Delta E_{it}<0\}} E_{it}/E_t < 1$ and thus the procyclical effect of the job-to-job transition rate $f_t \gamma$ is diluted when the job destruction rate is considered. Second, this share itself is countercyclical. Intuitively, when the total separation rate is considered, it includes a large number of procyclical job-to-job transitions that occur at expanding firms. These job-to-job separations are not part of the job destruction rate, and the countercyclical behavior of layoffs play a bigger role for the overall cyclicality of the job destruction rate. Note also that the flip side of this fact is that the job destruction rate is not the same as the layoff rate (EU transition rate) and is less countercyclical than the layoff rate (EU transition rate), because it is partially affected by the procyclical movements in the job-to-job transition rate. In the data, the job destruction rate is countercyclical but much less so than the EU transition rate. The mechanism discussed here explains the underlying reason for this empirical fact.

Lastly, note that in the model without job-to-job transitions, the job destruction rate is too volatile and its countercyclicality is too strong (see Table 5), whereas our benchmark

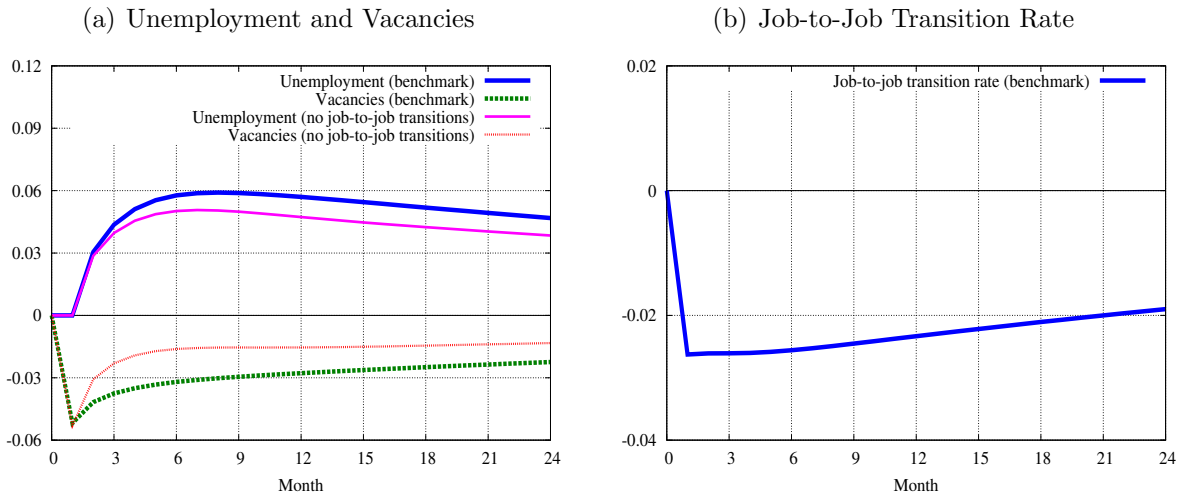

Figure 5: Impulse Responses of Unemployment, Vacancies, and Job-to-Job Transition Rate

Notes: Plotted are responses to a one-standard-deviation negative aggregate shock expressed as log deviations from the steady-state levels.

model does a much better job along both dimensions. The discussion in this section clearly demonstrates why this was the case. In the model without job-to-job separations, the only source of job destruction (at the monthly frequency) is EU separations. The job destruction rate is equivalent to the EU transition rate, which is more volatile and significantly more countercyclical than the observed job destruction rate.

6.6 Cyclicality of Unemployment and Vacancies

The last two rows of Table 5 present the cyclical statistics for unemployment and vacancies. Both models (with and without job-to-job transitions) generate a volatility of the unemployment rate that is too large relative to the data. For both models, this result comes from the EU transition rate that is too volatile, as already explained before.

In terms of the behavior of vacancies, our benchmark model performs much better. First, the volatility of vacancies is larger in our model and is closer to that in the empirical data. Second, vacancies are more procyclical in our model in line with the empirical data. In other words, job-to-job transitions play an important role in generating the Beveridge curve. This result is akin to the result reported by Fujita and Ramey (2012), who find a similar pattern in the single-worker matching models with and without job-to-job transitions. A novel feature of our multiple-worker model with job-to-job transitions is that job-to-job transitions significantly increase the volatility of vacancies relative to the model without job-to-job transitions. Panel (a) of Figure 5 plots unemployment and vacancy responses in the two models. One can see vacancies behave quite differently in the two models. In particular, our benchmark model generates vacancies that are significantly more persistent and volatile.

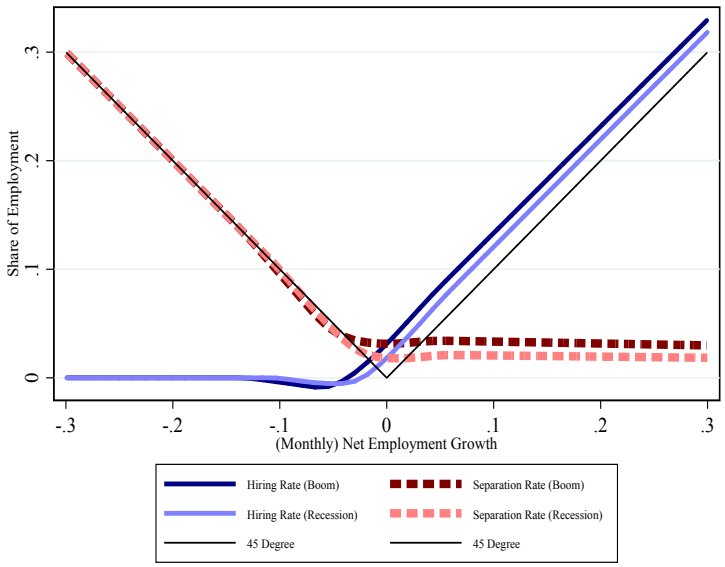

Figure 6: Hiring and Separation Functions in Boom and Recession Periods

Notes: Hiring and separation functions are based on the same 10,000 firms over the two different months, each of which corresponds to a boom and recession month. A boom (recession) month is an arbitrarily chosen period with a higher-(lower-)than-average TFP level.

The persistent declines in vacancies coincide with similar persistent declines in the job-to-job transition rate plotted in Panel (b).

The idea behind the result can be called the "vacancy chain," originally introduced by Akerlof et al. (1988). In Figure 6, we show how the hiring and separation functions differ in the boom and recession periods. The main differences in the hiring and separation functions exist on the right-hand side of the graph. That is, the hiring and separation functions both shift up (down) in the boom (recession) period. Note that the difference in the separation function reflects the strong procyclicality of job-to-job transition rate (recall that all separations at expanding firms are job-to-job transitions). This means that, *for the same level of net employment gains*, the firm needs to post more vacancies and thus hire more workers in the boom period, which is reflected in the upward shift in the hiring function. When a firm seeking to expand its employment loses workers to the other hiring firms, this firm needs to hire workers either from other firms or from unemployment. To the extent that this happens through poaching from other firms, it creates another chain of vacancy postings.

Importantly, Davis et al. (2012) empirically show that the "quit" function is highly sensitive to the business cycle conditions. This empirical result is highly consistent with our result here that the procyclicality of the job-to-job transition rate drives the business cycle movements of the hiring function.

6.7 Robustness

While our model is fairly tightly calibrated, it is worth checking whether the quantitative results are robust with respect to the variations of some of the exogenously chosen parameters. In particular, we considered the cases in which we set the curvature parameter of the production function α at 0.67, the persistence parameter of the idiosyncratic shock ρ_z at 0.9 and 0.99, and the elasticity parameter of the matching function at 0.6 instead of the values in the benchmark calibration. For each alternative value, we recalibrate the model following exactly the same procedure as in the benchmark calibration. We find that all the results under the benchmark calibration go through in those three cases. The results are available upon request.

6.8 Introducing a Firing Cost

In this section, we discuss the results from a simple extension of the model, namely, introduction of the firing cost. We adopt the simplest possible specification in which the cost per layoff is constant at τ.[31] Recall that, in our benchmark model, the job creation rate is weakly countercyclical, while in the data it is procyclical. Our earlier discussion in Section 6.5 suggests that one of the factors affecting the cyclicality of the job creation rate is the relative strength of the countercyclicality of the EU transition rate and the procyclicality of the job-to-job transition rate. Thus, introducing the firing cost can potentially shift this balance in the procyclical direction. In calibrating the model with the firing cost, we keep all the existing parameters at the same values as before and set the value of τ at 25% of monthly wage, which is fairly low and thus is in line with the fact that the firing cost in the U.S. is, on average, very small.[32] We also consider the case in which the firing cost is introduced into the model without job-to-job transitions. Again, we simply set the value of τ at 25% of the monthly wage without changing other parameter values.

Table 7 compares relative standard deviations and correlations with output in the four cases (models with and without job-to-job transitions and the firing cost). First, observe that in the model without job-to-job transitions, the introduction of the small firing cost slightly reduces the volatility of the EU transition rate and the unemployment rate (the volatility of the UE transition rate, on the other hand, increases slightly). Job creation and destruction rates become less volatile but remain too volatile relative to the empirical moments. Moreover, both of them remain strongly countercyclical. Overall, the cyclical properties of this model are largely insensitive to the introduction of the small firing cost.

More interesting interactions exist in the model with job-to-job transitions. In particular, the relative standard deviations of *all* transition rates increase with the introduction of the

[31] The optimality conditions presented in Section 3.4 needs to be modified in a straightforward manner with the introduction τ.

[32] We do not recalibrate the model for this exercise since recalibrating requires us to change the values of other parameters, making it difficult to isolate the effects of the introduction of τ. Because we do not recalibrate the model, introducing the firing cost has an effect of lowering the steady-state worker transition rates. However, its quantitative impacts are fairly small. The steady-state EU transition and UE transition rates fall to 1.3% and 23% from 1.5% and 25%, respectively.

Table 7: Effects of Firing Cost

	Relative SD				Correlation with Output			
Job-to-Job Transitions:	✓	✓	No	No	✓	✓	No	No
Firing Cost:	No	✓	No	✓	No	✓	No	✓
Worker transition rates								
EU transition rate	10.93	11.95	9.33	9.20	−0.85	−0.83	−0.75	−0.72
EE transition rate	5.21	5.96	0.00	0.00	0.99	0.98	0.00	0.00
UE transition rate	5.21	5.96	6.87	7.18	0.99	0.98	0.98	0.98
Job flow rates								
Creation rate	1.90	2.07	6.52	6.30	−0.19	0.07	−0.71	−0.67
Destruction rate	2.76	2.62	9.36	9.24	−0.48	−0.35	−0.74	−0.72
Stocks								
Unemployment rate	13.05	14.35	12.06	12.01	−0.89	−0.87	−0.89	−0.88
Vacancies	7.11	8.56	4.59	5.10	0.94	0.94	0.60	0.68

Notes: See notes to Table 5.

firing cost. The volatility of vacancies increase significantly by around 20% (as opposed to 10% in the model without job-to-job transitions). The underlying reason for this effect is that, when job-to-job transitions exist in the model, the *effective* cost of laying off workers is actually time varying (countercyclical), even though the cost per layoff is fixed at τ. This is because job-to-job transitions (quits) are not subject to the firing cost. Consider a firm i whose optimal employment decision is to reduce its employment size by $\Delta n_i (\equiv n_i - n_i')$. If $\Delta n_i > f n_i$ (i.e., the firm intends to reduce employment beyond quits), the cost per *separation* is $\tau(1 - f n_i / \Delta n_i)$; otherwise, it is zero. Because f fluctuates procyclically, this effective cost is countercyclical. The intuition is as follows: During a recession (boom), the pace at which the firm loses its workers through quits slows down (increases) and thus the firm needs to lay off more (fewer) workers for a given Δn_i. To verify this mechanism, we compute the average effective cost per separation of all shrinking firms in each period in our simulation and indeed find that this cost is highly countercyclical and volatile.[33]

The time-varying feature of this cost feeds back into the vacancy posting decision through the firm's dynamic considerations. Suppose that the firm receives a positive idiosyncratic shock in a recession. The firm knows that, with a lower job-to-job transition rate, the workers will stay longer at the firm, and the firm faces a higher (effective) cost of shedding workers should the firm receives a negative idiosyncratic shock in the future. This implies that the firm posts fewer vacancies in the current period (than in the case with no firing cost). The same positive idiosyncratic shock in a boom would strengthen the vacancy posting incentive, because the firm knows that shedding workers is less costly given that the job-to-job transitions would take care of them should the necessity arise.

An important dimension along which the larger volatility of vacancies (and thus that of the job-to-job transition rate) is helpful in improving the performance of the model is

[33]The standard deviation is larger than output volatility, and the correlation with output is −0.89.

the correlation of the job creation rate with output. Introducing the firing cost pushes the correlation coefficient significantly in the procyclical direction (from -0.19 to 0.07). While the procyclicality is still too weak, the exercise here demonstrates the usefulness of this extension.

7 Conclusion

In this paper, we have studied the quantitative properties of a multiple-worker firm matching model that features job-to-job transitions. We show that the model is capable of replicating the overall cyclical patterns of worker transition rates and job flow rates simultaneously. We demonstrate that the presence of job-to-job transitions plays a crucial role. The model without job-to-job transitions performs poorly in many dimensions.

One of the most important lessons of the paper is that although the job creation rate and the hiring rate appear to capture similar labor market flows, they behave quite differently from each other. Similarly, the job destruction rate and the separation rate also behave differently. In this paper, we thoroughly characterized these conceptual differences and show that the conceptual differences result in nontrivial differences in the observed cyclical behavior of these variables. We also find that our model with job-to-job transitions exhibits the "vacancy chain" whereby poaching of workers by one expanding firm prompts further vacancy postings at other expanding firms. This feature provides an important explanation for the observed persistence in job vacancies.

In developing our model, we made a few key simplifying assumptions, namely, zero worker bargaining power and the fixed search intensity of on-the-job seekers. With these assumptions, we were able to make significant progress on our issues of interest. However, adopting a more complete wage setting within our framework and endogenizing the search intensity are both extremely challenging, yet important extensions.

A Computation

Details of the numerical procedure to solve for the steady-state equilibrium and the dynamic stochastic equilibrium are as follows.

A.1 Steady-State Equilibrium

1. Guess equilibrium market tightness θ, which gives $q(\theta)$ and $f(\theta)$.

2. Guess $D(x, n')$, the expected marginal profit function of a firm with type (x, n'). Guess also the optimal employment adjustment rule $n' = g(x, n)$.

3. Use the first-order conditions (5) and (6) to obtain the (s, S) band, $\underline{n}^*(x, n)$ and $\overline{n}^*(x, n)$, of the firm's employment adjustment rule. These two functions are used to update the firm's optimal employment adjustment rule $n' = g(x, n)$.

4. Using the updated optimal employment adjustment rule $n' = g(x, n)$, (7), and (8), update the firm's marginal profit function $D(x, n')$.

5. Check convergence of $D(x, n')$ and $n' = g(x, n)$, based on the distance between the old and updated functions. If convergence is obtained, go to the next step. Otherwise, update $D(x, n')$ and $n' = g(x, n)$ and go back to Step 3.

6. Using the optimal employment adjustment rule $n' = g(x, n)$ and the stochastic process for x, simulate the economy until the invariant type distribution $m(x, n)$ is obtained.

7. Compute the total labor force (population) consistent with the stationary distribution m and the unemployment rate U as

$$L = \frac{\int g(x, n) dm}{1 - U}.$$

Note that U is fixed at a targeted level. In other words, we don't need to find an equilibrium U because we impose it to be exactly our target value. The efficiency-weighted number of searchers S, the total number of vacancies (normalized by the labor force) V, and the labor market tightness θ are calculated by

$$S = \gamma(1 - U) + U, \tag{15}$$

$$V = \frac{1}{L} \int \frac{\max[g(x, n) - (1 - \gamma f(\theta))n, 0]}{q(\theta)} dm, \tag{16}$$

$$\theta = \frac{V}{S}. \tag{17}$$

8. Check convergence of θ. If the distance between the guess and the updated numbers is smaller than the predetermined value, then stop. Otherwise, update θ and go back to Step 2.

A.2 Dynamic Stochastic Equilibrium

1. Set up the equilibrium forecasting functions. Note that the type distribution m is replaced by U. $\{U', \theta\}$ is the list of variables to be forecast through the following forecasting rules:

$$\log U' = \phi_0^1 + \phi_1^1 \log z + \phi_2^1 \log U \tag{18}$$

$$\log \theta = \phi_0^2 + \phi_1^2 \log z + \phi_2^2 \log U \tag{19}$$

Let Φ be the vector of the parameters $\{\phi_0^i, \phi_1^i, \phi_2^i\}_{i=1,2}$.

2. With a guess for Φ, and current z and U, we can predict $q(\theta)$, $f(\theta)$, and U'.

3. Guess the expected marginal profit of the firm with a type (x, n') under the aggregate state (z, U'), $D(x, n', z, U')$.

4. Using the first-order conditions (5) and (6), compute (s, S) band $\underline{n}^*(x, n, z, U)$ and $\overline{n}^*(x, n, z, U)$. These two functions characterize the firm's optimal employment adjustment rule $n' = g(x, n, z, U)$.

5. Using the optimal employment adjustment rule $n' = g(x, n, z, U)$ and the envelope condition (7), update the firm's marginal profit function $D(x, n', z, U')$ from (8).

6. Check convergence of $D(.)$. If the distance between the initial and updated $D(.)$ and $g(.)$ are smaller than a predetermined tolerance level, go to the next step. Otherwise, update $D(.)$ and $g(.)$ and go back to Step 3.

7. Using the optimal employment adjustment rule $n' = g(x, n, z, U)$ and the stochastic processes for z and x, simulate the economy for $T = T_0 + T_1$ periods. The economy consists of a panel of 1 million establishments over 1,320 periods with $T_0 = 120$ and $T_1 = 3,200$. The simulation starts with the steady-state distribution of $m(x, n)$. The unemployment rate in the initial period can be obtained from the distribution. The unemployment rate in each period is calculated as

$$U = \frac{L - \int g(x, n, z, U) dm(x, n)}{L}. \tag{20}$$

   The number of vacancies V, the number of job seekers S, and the labor market tightness θ in each period are calculated by the formulas (15), (16), and (17).

8. Using the sequence $\{z_t, U_t, \theta_t\}_{t=T_0+1,\ldots,T}$, run OLS regressions (18) through (19) and obtain the new set of coefficients Φ.

9. Check convergence of Φ. If the distance between the old and new Φ is smaller than a predetermined tolerance level, then stop. Otherwise, update Φ and go back to Step 2.

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